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Asymmetric Price Rigidity and the Optimal Rate of Inflation

Introduction

This paper addresses the issue of optimal rate of inflation. Stable inflation at some optimal level is a long-term goal of the monetary policy as opposed to short-term stabilization policy. In the paper it is argued that the optimal rate of inflation is positive due to a number of reasons. Essentially, the arguments in favor of positive optimal inflation fall into three groups. Desirable level of inflation may be positive as a result of optimization of the tax system (i.e., seigniorage argument). However, this does not seem to be an important reason for long-run monetary policy goals determination. The other two groups of arguments appear to be more relevant for long-run monetary policy. Positive rate of inflation is desirable for the financial markets and the conduct of short-run monetary policy since this assures that nominal interest rates be positive. And, finally, trend-inflation alleviates price stickiness and contributes to business cycle stabilization.

In this paper we investigate the relationship between long-run level of inflation and price stickiness. In particular we analyze the case of asymmetric price rigidity – a situation when prices are more rigid downward than upwards. The idea of asymmetric price rigidity is not new for (New) Keynesian theories. However, formal analysis of asymmetric rigidity is far from being exhaustive. It turns out that asymmetric price rigidity may enhance the optimality of a positive inflation.

In the first part of the paper we discuss briefly certain arguments on optimal rate of inflation. The second part of the paper is devoted to asymmetric price rigidity and it consequences. In the third part we present a formal model which is slight modification of Ball and Mankiw (1994) model. We conclude by indicating a number of possibilities to extend the current research.

Optimal Rate of Inflation

The issues connected with inflation have always been central to Macroeconomics. There is a wide consensus among the economists that high inflation affects negatively economic activity[1]. Two-digit inflation is virtually unacceptable for most developed countries.[2] At the same time monetary authorities of most countries seem to fear deflation much more than inflation of the equal size. This fear has recently become more obvious in the view of Japanese stagnation of the 1990's.

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[1] The mechanism of this phenomenon seems to be the following: higher inflation is always more volatile and unexpected; unexpected inflation distorts intertemporal decision and risk-averse agent cut down their transactions. One may think here of the Bruno and Fischer rule of 40% threshold level of inflation. *{I need a reference here}* The other reason outlined by Summers (1991) is that nonzero inflation generates an inefficient rent-seeking process for deferred payment.

[2] This may be not true, however, for developing countries, such as Russia, that face a substantial tradeoff between monetary stability and real growth. The real losses of bringing inflation down to one-digit level may be prohibitively high for these countries. Similar issues are discussed by Blanchard (2003).

There exists an implicit agreement among the central bankers that slightly positive (about 2 or 3 percent) stable inflation is desirable. For example, this point of view is advocated by Lawrence Summers (1991). However, Alan Greenspan declares that his long-term monetary goal is stable prices and zero inflation.[3] Nevertheless, very few countries with good macroeconomic dynamics show close to zero or negative rates of inflation. David Romer (2000) argues that since World War II most countries have witnessed only periods of disinflation but not deflation. This can be treated as an implicit evidence of the fact that most central banks target a somewhat positive long-term rate of inflation.

In theory there is no single leading point of view on this problem. The famous Friedman's rule (see, for example, Blanchard and Fischer 1989, section 4.5) states that the optimal rate of inflation should be negative and equal to minus real interest rate not to distort the consumers' decision of allocation of their assets between money and bonds. However, most economists do not take this theoretical result too seriously. Summers (1991) notes that money in the utility-function models that generate this result are "almost completely irrelevant". Moreover, the doctrine of monetarism, developed in a large scale by Friedman, suggests an optimal rate of money growth to be slightly higher than real growth rate in order to give some flexibility to the economy (see for reference Sachs and Larrain 1993, section 8.5).

On the other hand, there are a number of reasons in favor of a positive optimal rate of inflation. Essentially, there are three main groups of arguments. The first argument refers to the seignorage as a source of government revenue. Government tax optimization may imply positive inflation rates. However, the gains from optimal inflation tax in developed countries seem to be relatively small in comparison with overall monetary stability. As Summers (1991) argues, "optimal tax theory has little or nothing to do with sensible inflation policy". I support this point of view and, therefore, are not analyzing this issue in more detail.

Secondly, positive inflation is needed for the proper functioning of financial markets and monetary policy. Romer (1996) argues that the risk free real interest rate in the United States has been negative in about one third of the years since World War II. Low or zero rates of inflation would mean that the nominal interest rate would have to become very close to zero of even negative which could lead to a collapse of the financial system. Moreover, the stimulating short-run monetary policy is impossible given very low or negative interest rates (one may refer to this

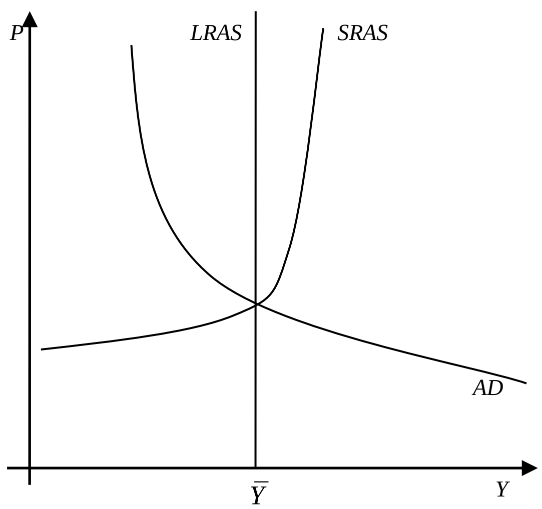
[3] Greenspan defines price stability as a rate of inflation that can be not taken into account when forming long-term intertemporal expectations. Inflation measured by CPI is biased upwards due to substitution effect and quality improvement. This bias is about 0.5-0.9%. According to Greenspan, Fed's long-term monetary goal is inflation within this interval. *{I do not have a direct reference for this. Lars Svensson told me that it can be found in the minutes of FRB meetings for 1996-99.}* At the same time, Summers (1991) argues that "proposed zero-inflation amendment is a good idea if it is not taken too seriously or too literally, but is instead viewed as a device for strengthening the independence of the Federal Reserve System".

situation as to Keynesian "liquidity trap"). The other point made by Fischer and Summers (1989) is that very low inflation targets by monetary authorities may be not time consistent.

And, finally, the third argument in favor of positive inflation is that it may function as a lubricant for the economy by making nominal prices more flexible. Empirical research by Ball, Mankiw, and Romer (1988) and Kiley (2000) suggests that higher rates of inflation do, in fact, make prices more flexible and business cycle less persistent. In particular, relative prices are becoming more flexible as well. In other words higher inflation alleviates real rigidities as well as nominal. This allows the economy to go through the business cycle more smoothly with smaller volatility in the output gap. This discussion does not take into account the possibility of the asymmetric price rigidity and money illusion which was often assumed in Keynesian theories. We develop this point in more detail in the following section.

Asymmetric Price Rigidity

Asymmetric price rigidity in Keynesian and New Keynesian literature implies that prices are more rigid in the downward direction than in the upward direction. In other words, prices can easily go up, while they resist going down. This phenomenon usually appears in the introductory level textbooks on Macroeconomics in the form of asymmetric (convex) short run aggregate supply curve (see pic. 1). In this case positive aggregate demand shocks lead to insignificant output increases and substantial increases in price level, while negative aggregate demand shocks on opposite reduce output significantly and do not affect prices too much.



Pic. 1. Asymmetric price rigidity in an AS-AD diagram.

The important consequence of asymmetric price rigidity is that the cost of business cycle becomes first order rather than second order since positive output gaps do not compensate for negative ones.[4]

The possible source of asymmetric price rigidity is labor market. Many economists (e.g., Tobin 1972 and Summers 1991) argue that nominal wages are extremely rigid downwards, while they can move upwards rather easily. In labor economics this phenomenon is called the *ratchet effect* and is a well studied empirical fact.[5] However, rational foundations for it are not yet developed, neither in Labor Economics nor in Macro. One may think of a money illusion of workers as a natural explanation for the ratchet effect. It is very likely that people are not completely rational and, hence, some money illusion is natural in real life.

In the New Keynesian literature there has been a number of works that tried to investigate the asymmetric price rigidity. A work by Timur Kuran (1983) builds upon a somewhat unnatural model. The asymmetry arises as a result of relationship between the discount factor and market growth rate of a firm that sets its price for two periods. Therefore, the asymmetry may be upward as well as downward conditional on the parameters values.

There is also a number of works in the spirit of Caplin and Leahy (1991) model, such as Caballero and Engel (1992). This models rest upon state-contingent pricing and optimal sS-rules for the firms. The asymmetry is generated in these models by either trend inflation or asymmetric distribution of shocks. However, this models are very technical, overcomplicated, and in most cases intractable. The work that seems to be the most fruitful in this field is the paper by Ball and Mankiw (1994). The asymmetry arises endogenously in this model as a result of trend inflation. However, this is a weak point of this model since the asymmetry cancels out once there is no trend inflation and even becomes negative for negative trend inflation. The Ball and Mankiw model will be analyzed more closely in the following section.

It is worth noting that that there have been some empirical works on asymmetric effects of shocks. Cover (1992) and Tsiddon (1993) find some empirical evidence in favor of asymmetric responses of output to aggregate demand shocks. However, Pete Klenow (2003)[6] investigates the firm-level micro data on price adjustment and concludes that prices are as flexible downward as upwards. Therefore, asymmetric rigidity is not a well established empirical fact yet.

Turning back to our main question of optimal inflation, it is worth noting that the intuitive way to model asymmetric rigidity is to introduce asymmetric menu cost which is higher for downward adjustment than to upward adjustment. A rational explanation for this intuitive

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[4] A good survey of literature on this topic is provided in Ball, Mankiw, and Romer (1988).

[5] See, for example, **Ronald Ehrenberg and Ronald Smith, 2000**, Modern Labor Economics, *7th eds., Addison Wesley Longman.*

[6] *{I could not find the reference yet}*

assumption is desirable. Unfortunately, it is very likely that one has to assume money illusion in order to get this result. However, both Tobin (1972) and Summers (1991) consider money illusion as a very plausible macroeconomic phenomenon.

Given a higher menu cost for downward adjustment the firms will bear higher costs if there are as many negative nominal (e.g., aggregate demand) shocks as positive ones. Therefore, positive rate of inflation is likely to reduce the costs for the firms and, hence, increase social well-being. Moreover, positive rate of inflation will render the prices more flexible and they will easier adjust to sectoral or firm-specific (idiosyncratic) shocks. Therefore, positive rate of inflation will contribute to smoothing the business cycle and reduce overall output variability. This affects may be seen in a model of the following section.

The Model

In this section we present a slight modification of the Ball and Mankiw (1994) model which takes into account the possibility of asymmetric price rigidity. The asymmetry is introduced into the model exogenously by simply setting different menu costs for upward and downward price adjustments (downward nominal price adjustment is more costly than upward). A separate model is needed in order to give rational foundations for the asymmetric menu costs. However, here the main objective is to look at the consequences of possible asymmetric price rigidity for the general equilibrium of the model.

The set up of the model is the following. The aggregate demand (in logs) in the economy is

$$y_t = m_t - p_t, \tag{1}$$

where m is the money supply and p is the aggregate price level. The money supply follows a random walk with drift

$$m_t = \pi + m_{t-1} + \theta_t, \tag{2}$$

where π is "trend" money growth[7] and θ_t is a random zero-mean money supply shock at t independently and identically distributed (iid) according to cumulative distribution function (cdf) $F(\theta)$.

There is a continuum of firms characterized by their menu cost C, which is distributed according to cdf $G(C)$:

$$\int_0^{\bar{C}} dG(C) = 1.$$

[7] Note that trend money growth and trend inflation are essentially the same since the economy will convergence eventually to its flexible price equilibrium where all money supply shocks translate one-for-one into price level changes. Therefore, inflation and money growth may differ in the short run, but not in the long run.

All firms are engaged in monopolistic competition in a market of diversified product. The optimal nominal price for each firm is simply the current money supply

$$p_t{*} = m_t = m_{t-1} + \pi + \theta_t \,. \tag{3}$$

If not for the menu cost, all firms would set their nominal prices at the optimal level. Therefore, potential output (defined as the level of output that corresponds to absolutely flexible prices) in the model is zero:

$$\bar{y} = m - p{*} = m - m = 0 \,.$$

The firm's loss from non-optimality of its price is quadratic, which may be interpreted as a second order approximation for a general loss function (see, for example, BMR):

$$Loss_t = \left(q_s - p_t{*}\right)^2 , \tag{4}$$

where q_s is the price set by the firm in period s (the price setting process described below is such that s can be either t or t–1). For convenience we denote by x_s the following expression

$$x_s = q_s - m_s \,, \tag{5}$$

where x_s may be interpreted as a relative price set by the firm. Hence, the loss of the firm in period t=s (the period of price resetting) is simply x_t^2; the loss of the firm in period t=s+1 is $\left(x_{t-1} - \pi - \theta_t\right)^2$.

The crucial feature of the model is the price setting process, which combines both time and state contingency in price adjustment. A representative firm sets the price every two periods just like in Taylor model. However, its commitment to maintain the price fixed in the second period is not absolute. The firm may make an extra adjustment in the second period by paying the menu cost. Clearly, the firm will do so if there is a large shock to its optimal price, so that the losses from non-optimality of its fixed price exceed the menu cost. Therefore, the model inherits the simplicity and tractability of time-contingent models (such as Taylor or Fisher), but it also allows for asymmetric reaction of the firm to different shocks just like in state-contingent models (e.g., Caballero and Engel).[8]

As it was shown by Ball and Mankiw, this model endogenously generates asymmetric price rigidity given non-zero trend inflation ($\pi \neq 0$). When trend inflation is zero, the asymmetry of price rigidity vanishes from the model. Moreover, zero inflation is optimal from the social welfare standpoint. In this paper we challenge this result by introducing (exogenously) an asymmetric rigidity into the model. The firm has to pay a higher menu cost in order to reset its price downwards. If the menu cost for upward adjustment is C, the menu cost for downward adjustment

[8] Clearly, there is no place for asymmetric price adjustment in pure time-contingent models. In such models a firm resets its price after a fixed or random number of periods and resets its price optimally. Therefore, there may be no asymmetry. On the other hand, pure state-contingent models lack tractability and there exhaustive analysis is impossible (Blanchard and Fischer).

is αC, where $\alpha > 1$. Hence, the firm will compare $Loss_t = (x_{t-1} - \pi - \theta_t)^2$ with C if $\theta_t > x_{t-1} - \pi$ and with αC otherwise.

Now let's turn to the firm's problem when it sets the price. We analyze the optimal decision of a firm with an arbitrary menu cost *C*. At period *t*–1 after the realization of the shock θ_{t-1} the firm sets x_{t-1} in order to minimize its two period expected losses (there is no discount factor for the second period). In the second period the firm will choose to reset its price if $\theta_t > x_{t-1} - \pi + \sqrt{C}$ or $\theta_t < x_{t-1} - \pi - \sqrt{\alpha C}$. Clearly, it will set its relative price to a new optimum $\pi + \theta_t$ (in other words, its nominal price will equal $p_t{*} = m_{t-1} + \pi + \theta_t$). Denote by $\underline{\theta}$ and $\bar{\theta}$ the upper and lower thresholds values of θ correspondingly:

$$\begin{aligned} \underline{\theta} &= x_{t-1} - \pi - \sqrt{\alpha C}, \\ \bar{\theta} &= x_{t-1} - \pi + \sqrt{C}. \end{aligned} \tag{6}$$

Already at this stage one may see the asymmetry in the optimal response of the firm to different shocks. Firms will adjust to a much smaller positive shocks than negative shocks given $x_{t-1} > 0$, $\pi > 0$ and $\alpha > 1$. If we keep $\alpha > 1$, this asymmetry will remain even when $x_{t-1} = \pi = 0$. This feature of the model makes it fundamentally different from that of Ball and Mankiw.

The firm's problem in period *t*–1 may be formalized as

$$x_{t-1} = \arg\min_x \left\{ x^2 + \int_{\underline{\theta}}^{\bar{\theta}} (x - \pi - \theta)^2 dF(\theta) + C\left(1 - F(\bar{\theta})\right) + \alpha C F(\underline{\theta}) \right\}, \text{ given (6).}^9 \tag{7}$$

The first term in (7) reflects the loss from non-optimality of the price in period *t*–1 when the price is just set for two periods. The second term is the expected loss from non-optimality of the price in the second period given that the firm does not adjust its price.[10] And, finally, the third and the fourth terms stand for the menu cost borne by the firm when it increases and decreases its price respectively.

The solution to problem (7) will depend on the parameters of the problem α, π, and *C*

$$x_{t-1} = x^*(C \mid \alpha, \pi), \quad \underline{\theta} = x^*(C \mid \alpha, \pi) - \pi - \sqrt{\alpha C}, \quad \text{and} \quad \bar{\theta} = x^*(C \mid \alpha, \pi) - \pi + \sqrt{C}.$$

While α and π are the same for all the firms, *C* is specific to each firm. Non-trivial distribution of *C* assures that for each shock θ_t some firms will bear the menu cost and reset the price and some will remain with their old price contributing to price stickiness in the economy.

[9] Note that (6) already constitutes optimally chosen values of $\underline{\theta}$ and $\bar{\theta}$. More generally, one could omit restrictions (6) and optimize (7) with respect to *x*, $\underline{\theta}$, and $\bar{\theta}$. This will give the same results.

[10] To be more specific, it is conditional expectation multiplied by the probability of the condition. In other words, it is the expected loss from non-optimality multiplied by the indicator function that equals one when the firm does not adjust.

The formal solution of (7) gives[11]

$$x^* = \frac{1}{1+F(\bar{\theta})-F(\underline{\theta})}\left[\pi\left(F(\bar{\theta})-F(\underline{\theta})\right)+\int_{\underline{\theta}}^{\bar{\theta}}\theta dF(\theta)\right]. \qquad (8).$$

Along with (6), (8) gives us a system of three equations with three unknowns. Unfortunately, it is impossible to get a general closed form solution for this problem. However, one can find a numerical solution for specific distribution of θ (e.g., normal distribution).

The expression (8) is very intuitive. The firm sets its price in period *t*–1 as a weighted average of the (expected) optimal prices in period *t*–1 and t, where the weights are the probabilities that the price set by the firm will be in effect in the respective period. The optimal (relative) price in period *t*–1 is zero, while the probability that x_{t-1} will be in effect in this period is one. Further, the optimal price in period *t* is $\pi+\theta_t$. The conditional expectation of $\pi+\theta_t$ given that x_{t-1} is in effect is

$$E\left(\pi+\theta_t \mid \underline{\theta}<\theta_t<\bar{\theta}\right)=\pi+\int_{\underline{\theta}}^{\bar{\theta}}\theta dF\left(\theta \mid \underline{\theta}<\theta<\bar{\theta}\right)=\pi+\int_{\underline{\theta}}^{\bar{\theta}}\frac{\theta dF(\theta)}{F(\bar{\theta})-F(\underline{\theta})},$$

while the probability of this event is $F(\bar{\theta})-F(\underline{\theta})$. This result squares well with the fact that firms have quadratic loss functions. This implies that they set their prices as certainty equivalents, which is exactly what we observe in our case. Thus, the assumption of the quadratic loss function may be very restrictive. It is interesting to assess the robustness of the model to the relaxation of this assumption. However, the absence of certainty equivalence in price setting will render the model intractable and it will be impossible to solve it even numerically. We will turn back to other consequences of certainty equivalence in price setting later on in the paper.

From (8) one can see that x^* is likely to be less than $\frac{\pi}{2}$ for a wide range of *C* and $F(\theta)$, since $F(\bar{\theta})-F(\underline{\theta})<1$ and $E\left(\theta_t \mid x_{t-1}-\pi-\sqrt{\alpha C}<\theta_t<x_{t-1}-\pi+\sqrt{C}\right)$ should be slightly negative due to asymmetry in price rigidity.[12] As argued above, this means that firms will adjust to much smaller positive shocks than negative shocks. Therefore, negative monetary shocks will contribute much more significantly to output declines than positive monetary shocks to output increases. In view of this, the idea that output gains and losses from business cycle generated by positive and negative aggregate demand shocks is of second-order becomes very questionable. It is worth noting, that for small trend inflation ($\pi \approx 0$) x^* will be negative, which contributes even more to the asymmetry.

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[11] This can be obtained via direct partial differentiation of the objective function in (7) with respect to *x*. According to the envelop theorem, there is no need to take the full derivative and differentiate with respect to $\underline{\theta}$ and $\bar{\theta}$ since they are already chosen optimally. However, one may check it directly. The first order condition is sufficient for this problem since it is convex.

[12] …Example with normal distribution…

As a result, $|\underline{\theta}|>\overline{\theta}>0$, which implies that firms would adjust to smaller positive shocks than negative.

We also note that $x^*(C\,|\,\alpha,\pi)$ is not monotonic in π. If π becomes very high, the firm will more often adjust its price in the second period and, hence, it will set x^* closer to zero. Let's now compare x^* with the optimal two-period relative price that would arise in Taylor model, where firms do not have the option to reset their prices in the second period.[13] Clearly, it would be

$$\frac{1}{2}\left[\pi+\int_{-\infty}^{+\infty}\theta dF(\theta)\right]=\frac{1}{2}\left[\pi+E\theta_t\right]=\frac{\pi}{2}>x^*.$$

This means that firms will set the price in the first period lower than they would in the Taylor model. Therefore, initially the output will be higher and this may partially compensate for the adverse effects of asymmetric price rigidity in the second period. We shall return to this questions later on.

Now it is time to look at the general equilibrium of the model for any given α and π. As in Taylor model it is assumed that half of all firms have their scheduled price changes in odd periods, while the other half does it every even period. We need to find the aggregate price level in the economy for any realization of θ in the second period.[14] Given the money supply, the knowledge of the aggregate price level allows us to obtain the stochastic path of the output in the economy.

The aggregate price level in the economy in period t is

$$p_t(\theta)=m_{t-1}+\frac{1}{2}\left[(\pi+\theta)G(\hat{C})+\int_{\hat{C}}^{\overline{C}}x^*(C)dG(C)+\int_{0}^{\overline{C}}\left(x^*(C)+\pi+\theta\right)dG(C)\right], \qquad (9)$$

where $\hat{C}$ is the threshold level for the value of menu cost: all firms with $C\leq\hat{C}$ will reset their prices given shock θ in the second period, while firms with $C>\hat{C}$ will keep their old prices. Hence, $\hat{C}$ solves the following equation:

$$\begin{array}{l}\hat{C}=\left(x^*(\hat{C})-\pi-\theta\right)^2\\ \alpha\hat{C}=\left(x^*(\hat{C})-\pi-\theta\right)^2\end{array},\quad \text{for}\quad \begin{array}{l}\theta>0\\ \theta<0\end{array}. \qquad (10)$$

Clearly, smaller number[15] of firms will adjust to negative shocks. Equation (10) shall be solved for every θ to obtain $\hat{C}(\theta)$.

Let's now interpret the elements in (9). The aggregate price level at t will be composed of pricing decisions of the firms in period t–1 and t. These pricing decisions already take into account all monetary shocks up to t–1. In other words, firms pricing decisions will rest upon the value of

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[13] Note that Taylor model is a special (limiting) case of our model when $C\to\infty$. This implies that firms will never reset their prices in the second period.

[14] This stochastic equilibrium will remain in every period, since the problem of the firms is dynamically consistent (in the sense that its solution does not change with time).

[15] Smaller measure of firms, to be more precise.

m_{t-1}. This explains the first term in (9). Note that this means that all shocks will have effect on the economy only for one period.[16] We turn now to the elements in square brackets in (9). The factor of one half before the square brackets reflects the fact that there are two types of firms: those that set prices at t–1 and t. The first term in square brackets corresponds to the firms that set price t–1 and reset at t: they choose the optimal price $\pi+\theta$ and the measure of such firms is $P\{C \le \hat{C}(\theta)\} = G(\hat{C})$. The second term corresponds to the firms that set the price in t–1 but do not reset it in t. And the final term reflects the price set by the firms at t.

Given $p_t(\theta)$, we may find now $y_t(\theta)$:

$$\begin{aligned} y_t(\theta) &= m_t - p_t(\theta) = \pi + \theta - \frac{1}{2}\left[(\pi+\theta)G(\hat{C}) + \int_{\hat{C}}^{\bar{C}} x^*(C)dG(C) + \int_0^{\bar{C}} (x^*(C)+\pi+\theta)dG(C)\right] = \\ &= \frac{1}{2}(\pi+\theta)\left(1-G(\hat{C})\right) - \frac{1}{2}\left[\int_{\hat{C}}^{\bar{C}} x^*(C)dG(C) + \int_0^{\bar{C}} x^*(C)dG(C)\right]. \end{aligned} \tag{11}$$

Expression (11) is our final result. Unfortunately, there exists no closed form solution for output. However, one can numerically obtain the distribution of output given the distributions of monetary shocks and menu costs.

Let us consider two special (limiting) cases now: the case of no menu cost (pure flexible prices) and the case of infinite menu cost (pure time-contingent pricing). If there were no menu cost ($C \equiv 0$), each firm would (re)set its price to the optimal every period (i.e., $x^* = 0$ and $G(\hat{C}) = 1$). As a result, the output would always be at its potential, flexible-price, level. With infinite menu cost ($C \equiv \infty$) our model, as discussed above, transforms into Taylor model with no option for firm to reset their prices in the second period. In this setting the firms would set $x^* = \frac{\pi}{2}$ (which is average expected optimal price) and the share of firms resetting the price in the second period would be zero (i.e., $G(\hat{C}) = 0$). In this case, output equals $\frac{\theta}{2}$, which reflects the fact that half of the firms won't be able to adjust to the shock.

An important general result that one may expect to obtain here, is that expected output (with respect to distribution of shocks) will equal zero:

$$Ey = \int y(\theta)dF(\theta) = 0. \tag{12}$$

This happens by virtue of two features of the model: the firms set their prices as certainty equivalents (i.e., the price set by a firm is an average of expected optimal price) and the problem of a firm is dynamically consistent. This result might seem rather surprising since we have asymmetric price rigidity in the model: the output increases less in response to positive shocks than it decreases

[16] This happens due to oversimplification of the assumption of the model concerning the optimal price setting which implies no strategic complementarity and, hence, no staggering. However, this is not the primary interest of the model.

in response to equivalent negative shocks. The intuition behind this result is the following: forward-looking firms initially set the price lower than they would in the model with symmetric price rigidity (when $\alpha = 1$ and $\pi = 0$); this pushes the level of output up so that on average output still equals zero.

This result may be called the strong form of natural rate hypothesis: the output will stay on its natural rate not only in the long run, but also it will on average equal the natural rate at every single period. If we relax the assumption that gives us certainty equivalence in the price setting (i.e., the quadratic loss function), we, probably, won't get this beautiful result.[17]

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Finally, we turn to the question of socially optimal rate of inflation. To answer this question we first need to define how the social welfare is measured. We will assume that social welfare is inversely proportional to average firm's losses (i.e., the value of the objective function of a firm – see equation 7). This may be an oversimplifying assumption since social welfare clearly depends on the volatility of output, which we do not account for. There are two reasons to take this assumption. Firstly, firms are the only agents in the model. To include consumers into the measure of social welfare, we will need to complicate the model by introducing consumer's utility function and decision making process. Secondly, this assumption was taken in the original paper by Ball and Mankiw, and it will be desirable to compare the results.

As a first step, we find the optimal rate of inflation for a firm with menu cost *C*. To find it, we substitute the optimal decision of the firm into its loss function:

$$Loss = (x^*)^2 + \int_{\underline{\theta}}^{\bar{\theta}} (x^* - \pi - \theta)^2 dF(\theta) + C\left(1 - F(\bar{\theta})\right) + \alpha C F(\underline{\theta}). \quad (13).$$

Now our goal is to find the minimal value of loss function with respect to inflation rate. Clearly, this problem should have an internal solution since both very large positive and negative inflation is bad for the firm (it always bears the menu cost which is its guaranteed minimum). The first order condition for the minimization problem is presented below:[18]

$$\int_{\underline{\theta}}^{\bar{\theta}} (\pi + \theta - x^*) dF(\theta) = 0. \quad (14)$$

Clearly, zero inflation will not deliver minimum to the problem since the asymmetric rigidity remains in our model even when there is no trend inflation. Given zero trend inflation ($\pi = 0$), the firm's optimal price would be

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[17] Unfortunately, I cannot provide so far the formal proof of this result. However, simulations suggest that at least this result holds for some important special cases (i.e., normal distribution of monetary shocks and uniform distribution of menu cost).

[18] To compute the F.O.C. one should take only partial derivative with respect to π. This is true by virtue of envelope theorem: all other elements (derivatives of x^*, $\underline{\theta}$, and $\bar{\theta}$) will cancel out due to the F.O.C. in the firm's problem.

$$x^* = \frac{1}{1 + F(\overline{\theta}) - F(\underline{\theta})} \cdot \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta) < 0,$$

since $|\underline{\theta}| > \overline{\theta}$ due to the asymmetric menu cost[19] and the distribution of θ is by our initial assumption. This implies that

$$\left.\frac{dLoss}{d\pi}\right|_{\pi=0} = \left[1 - \frac{F(\overline{\theta}) - F(\underline{\theta})}{1 + F(\overline{\theta}) - F(\underline{\theta})}\right] \cdot \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta) < 0,$$

which in its turn suggests that the optimal rate of inflation is positive.

To find the optimal rate of inflation we recall the first order condition for the firm's problem:

$$x^* = \int_{\underline{\theta}}^{\overline{\theta}} (\pi + \theta - x^*) dF(\theta). \tag{15}$$

Combining (14), (15), and the firm's optimal price (8), yields the following simple result. The optimal rate of inflation for a firm with menu cost *C* is such that the firm sets its first period price to zero:

$$x^*(\pi^*) = \frac{1}{1 + F(\overline{\theta}) - F(\underline{\theta})} \left[\pi\left(F(\overline{\theta}) - F(\underline{\theta})\right) + \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta)\right] = 0. \tag{16}$$

This means that the optimal rate of inflation is

$$\pi^* = -\frac{1}{F(\overline{\theta}) - F(\underline{\theta})} \cdot \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta) = -E\left(\theta \mid \underline{\theta} < \theta < \overline{\theta}\right) > 0.^{20} \tag{17}$$

The intuition behind this result is rather straightforward. Positive rate of inflation will allow the firm to adjust more frequently to positive shocks than to negative shock, which is less costly by our assumptions. Moreover, the firm won't have to bear the costs of non-optimality of its price in the first period.

After we have found the optimal rate of inflation for a single firm with menu cost *C*, we turn back to the question of socially optimal rate of inflation. In the model the firms are heterogeneous and differ in their menu costs. Therefore, the optimal rate of inflation will be different for different firms. However, it is positive for every firm, which implies that the socially optimal rate of inflation will also be positive.[21]

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[19] The firm will set negative first period relative price in order to adjust to negative shocks more rarely not to bear the higher menu cost. The limiting case is the Ball and Mankiw model with $\alpha = 1$, where zero inflation was optimal for all firms.

[20] Note that this is not a final result yet, this is only an equation from which one may find optimal rate of inflation as a function of the firm menu cost: $\pi^*(C)$.

[21] For example, one may think of a social welfare function in this model as a sum of firm's welfare across all firms.

What has to be done

In this section I will briefly outline a number of things that still need to be done in this field. First of all, in order to have a well-specified equilibrium model one needs to find some theoretical foundations for asymmetric menu costs and asymmetric price rigidity. It is very likely that this asymmetry may arise as a result of two reasons: positive trend inflation or money illusion. The first reason seems to be unsatisfactory since the asymmetry will cancel out from the model once the trend inflation is reduced to zero. Moreover, the model will generate an opposite asymmetry given negative trend inflation. This seems to be rather counterintuitive. However, it is a good testable implication.

The assumption of money illusion is not very welcome in macroeconomics. It is very desirable to find a possible rational foundation for money illusion. However, this seems implausible. Therefore, the possible suggestion would to provide some empirical evidence on the existence of the money illusion. Another interesting development of this work is to incorporate the labor market and model it explicitly with a downward nominal wage rigidity.

One of the most challenging issues is to provide some solid empirical evidence in favor of (or against) asymmetric price rigidity and output response. It would be interesting to compare the real-economy output responses to aggregate demand shocks to that generated in the model.

This research may be also extended to the analysis of sectoral (firm-specific) shocks. I believe this will enhance the effects of asymmetric price rigidity on output distribution.

And, finally, one can ask a more general question about the long-run optimal rate of inflation. It is interesting to evaluate relative significance of the arguments in favor of positive inflation (for example, in terms of output losses during the business cycle). However, this seems to be a very ambitious task. Another challenging question is to investigate whether the level of steadye inflation may have some real effects for the economy and whether there are some internal forces in the economy that resist inflation to become negative.

Conclusion

As we have argued, there is a number of reasons why optimal long-run inflation should be positive. Unfortunately, there is no single model that could capture all costs and benefits from stable inflation in order to find global optimum. In this paper we investigate the effect of trend inflation on price stickiness and business cycle volatility under asymmetric price rigidity.

The model presented in the paper suggests that the socially optimal level of trend inflation is positive given asymmetric price rigidity. However, long-run inflation does not affect average output, though it contributes to business cycle stabilization.

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