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RELATIONSHIP BETWEEN INTEREST RATE AND CORPORATE BOND YIELD[1]

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Abstract

The author created a model that describes the relationship between the current bank interest rate (rate on loans extended to business entities) and future corporate bond yield (in the text this is formula # 17): $\mathbf{C_{bank} = (k+C_{bond})/(1\text{-}r)}$. Where: $\mathbf{C_{Bank}}$ is interest rate on bank loans; $\mathbf{C_{Bond}}$ is bond yield; **r** is yield anticipated by shareholders; **k** is special ratio calculated from the formula # 22 which depends on authorized capital, EBIT, depreciation and income tax rate. Use of the model applied to Russia (calculations of early 2010) has shown that to the beginning of 2011, the financial situation in raw material industries will improve, while in other industries it will aggravate.

1. Introduction

The objective of this paper has been to derive a formula that describes relationship between the current bank percentage rate (rate on loans extended to business entities) and future corporate bond yield.

Scientific literature of today may offer a great deal of satisfactory studies of the relationship between the bank interest rate and various economic and financial parameters including its connection to yield on the bond market.

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For instance, it is to be noted that the central banks in many countries show their strong professional interest in practical aspects related to the bank percentage rate. Therefore, researchers focus their attention on such applied issues as relationship between inflation values and the yield of zero-coupon treasury bonds (Vineer Bhansali, Mark B.Wise, 2006) or estimated future inflation value from the bond yield curve (Jeffrey A.Frankel, Cara S. Lown, 1991).

In terms of financial markets, the bank interest rate may be regarded as the basis for estimation of resources in financial markets (Fadil Govori, 2007). This makes the interest rate a very important tool for both appraisal of financial market efficiency and management and operation on the markets. For instance, the model of corporate bond yield (Steven C.J.Simon, 2006) takes account of and diversifies the risk of interest rates.

Many researchers indicate relationship between the bank interest rate and bond yield, for instance, that current short-term interest rates and the current bond yield rate are in linear relationship (Natalia Ilieva, 2009).

The relationship between the bank rate and bond yield is often studied using statistical methods, in particular, correlation of two data series. This serves as a basis for mathematical models and conclusion that correlation is available or not.

The author decided not to use statistical approaches but to focus his efforts on search for such relationship between parameters which are logically substantiated. To do so, he made a few assumptions that simplified the review of behavior of parties to a loan contract and enabled him to describe the relationship as a mathematical model.

At the beginning of the process the generation of the bank interest rate at strong limitations to raise funds is reviewed. In particular, an assumption is made that financing of an investment project is possible only due to a bank loan, otherwise the project is unfeasible.

Further, the analysis covers a wide range of the project financing alternatives: own funds, a bank loan, bond and share issue. Chiefly, attention is focused on how a choice is made between two financing sources: a bank loan and bond issue. Also, the formulae making the model of relationship between the bank rate and bond yield are explained. Next, the author theoretically studies the resultant model and, finally, he considers application of the formula. All the calculations are based on the Russian environment.

2. Limited alternatives: either to sign a loan contract or abandon the project

2.1. The work underlying considerations:

1. Lending is the only source of bank income.
2. Bank loans comprise the only source of investments for businesses.
3. There is only one commercial bank.
4. Bank's shareholders show interest only in the bank's financial gains rather than in use of the bank for other purposes, for instance, for the purpose of their public image or provision of indirect support of other businesses, or black-market or criminal transactions.
5. There is only one borrower.
6. A borrower applies for a loan to finance a new investment project, satisfactory and of positive net present value (NPV). In addition, the borrower provides sufficient property security.
7. Both the bank and the borrower take reasonable actions.

In other words, banks may earn profit only due to lending, while businesses may receive financial resources only from banks. Also, there is no competition between banks to obtain a customer and there is no interbank loan market.

Besides, make a few more assumptions which will be needed later:

8. Bond issue takes about one year.
9. Share issue takes about two years.
10. Financial markets are characterized by strong competition between finance suppliers for customers.

2.2. Reasoning

So, there is only one bank and only one borrower on the market. According to the predetermined conditions they have to strike a deal otherwise both of them will be losers: the bank will not make a profit on money lent, while the business will not gain a profit on implementation of the investment project. Nevertheless, either of them has an alternative: either to strike the deal or not, and plus own logic in making a decision. Now we shall review how each of them behave when, as has been mentioned before, the bank and the borrower are singular.

2.2.1. First, we shall review actions of the bank

The bank will lend money, if the interest rate on the loan brings profit to the bank. It will not lend money, if the interest rate does not cover the bank's expenses and does not bring it a profit. But if the bank fails to lend money, it will result in the bank's direct loss as bank maintenance cost. Consequently, the bank is motivated by intention to determine a minimum interest rate (R_{min}) to offer to the borrower below which lending a loan will bring it no profit.

Two points are noteworthy. The bank will not reduce the rate below the minimum value proceeding from the premise that if there is no opportunity to earn profit, then the bank will minimize necessary damage. I think that the reason is subjective, in particular: since a loan will bring the borrower profit, bank's managers are not willing to take part in the project which improves welfare of the partner and deteriorates the bank's capital.

The bank will not decrease the rate below the minimum value proceeding from the premise that if it saves business (though it appeared to be unprofitable in the current period), in the next period it will lend a loan on such beneficial terms so as to cover the loss in the current period and ensure the average yield in the two periods in amount equal to the mid-market yield. The fact is this scenario is unlikely and banks do not assume that high risks.

2.2.2. Now we shall review actions of the business

The business, being profit-oriented, i.e. to implementation of the investment project, desires to receive a loan, since the project may not be realized without such loan. But if the interest rate exceeds a specified value (R_{max}) when the investment project is not attractive any more, the borrower shows no interest in the project and abandons it and, consequently, refuses the loan.

So, participants in the loan market deal with two borderline values of interest rate: the minimum one at which the bank lends a loan (R_{max}) and the maximum one at which a business raises a loan (R_{max}). In normal operating conditions of the banking sector the maximum business's rate is to outweigh the minimum bank's rate. The actual rate at which the parties sign a loan contract will lie between minimum and maximum rates and rely upon negotiations outcome.

Then the traditional view of the purpose of money authorities as to interest rate is to be put aside. It is generally agreed that authorities have to stimulate the financial sector to reduce interest rates on bank loans. However, taking account of two boundary rate values we suppose that there is another more reasonable purpose of the governmental regulation: to widen the range between the minimum bank's rate and maximum business's rate. Widening of the spread widens the trading interest rate range and, hence, makes conclusion of a loan contract more probable.

It is noteworthy, that if competition between banks for customers is not ideal, the actual interest rate on loans will be near to the upper limit of the range.

2.3. Boundary values of interest rate

Let us consider components of: the minimum bank's rate 1.3.1 and maximum business's rate 1.3.2.

2.3.1. The minimum bank's rate is made up of five components ensuring banking business: a) cost of funds lent by a bank; b) expenses for compulsory reserving of raised funds; c) bank maintenance costs; d) minimum cost efficiency below of which shareholders are unwilling to run banking business; and e) special-purpose deductions to the insurance fund in the event of the borrower's default:

$$R_{\min} = R_a + N + R_b + R_c + R_d \qquad [1]$$

- $\mathbf{R_{min}}$ is minimum interest rate at which the bank is prepared to lend money to the borrower;
- $\mathbf{R_a}$ is cost of funds;
- **N** is expenses for compulsory reserving of raised funds;
- $\mathbf{R_b}$ is bank maintenance costs;
- $\mathbf{R_c}$ is minimum cost efficiency anticipated by the bank's shareholders;
- $\mathbf{R_d}$ is insurance margin.

Now we shall consider each component of the minimum bank's rate.

Cost of funds comprises a weighted sum of costs of resources available to the bank. The amount is described by the following formula:

$$R_a = W_1 R_1 + W_2 R_2 + W_3 R_3 + W_4 R_4 + W_5 R_5 + W_6 R_6 \qquad [2]$$

- $\mathbf{R_a}$ is cost of funds;
- **W** is weight of a component in shares;
- $\mathbf{R_1}$ is cost of the bank's own funds;
- $\mathbf{R_2}$ is cost of funds raised on the interbank loan market;
- $\mathbf{R_3}$ is cost of funds acquired from the Central Bank;
- $\mathbf{R_4}$ is cost of deposits placed with the bank by legal entities;
- $\mathbf{R_5}$ is cost of deposits placed with the bank by private individuals;
- $\mathbf{R_6}$ is cost of balances in accounts of customers.

Formula 2 may have some more components, for instance, it may include items related to raising funds in deposit auctions under auspices of the Ministry of Finance or unsecured auction held by the Central Bank. The final configuration of this formula is dictated by forms and types of funds attracted by the bank.

Expenses on compulsory reserving of raised funds rely on requirements of the Central Bank. In the simplest case, this may be a flat rate for various sources of funds except account balances.

Bank maintenance costs. This factor reflects office rentals, staff pay-roll and other expenses that ensure functioning of the bank:

$$R_b = \frac{S_b}{S_a} \qquad [3]$$

- $\mathbf{R_b}$ is bank maintenance costs;
- $\mathbf{S_b}$ is absolute total of expenses for bank maintenance for the period;
- $\mathbf{S_a}$ is total of loans lent in the period.

Minimum cost efficiency anticipated by the bank's shareholders directly depends on alternative investments and may not be below of the mid-market yield. Otherwise, it may result in that shareholders will be willing to quit the banking business and invest into some other area (We would like to remind that in compliance with assumption 4 the bank's shareholders are entirely interested in equity growth). This parameter depends on the ratio between the dividends anticipated by the shareholders and invested capital:

$$R_c = \frac{D}{C}, \qquad \text{where } R_c \geq R_r \qquad [4]$$

- $\mathbf{R_c}$ is minimum cost efficiency anticipated by the bank's shareholders;
- **D** is absolute total of dividends anticipated by the bank's shareholders;
- **C** is bank's equity;
- $\mathbf{R_c}$ is mid-market yield of alternative investments of the same risk category.

Special-purpose deductions to the insurance fund in the event of the borrower's default comprise a derivative of default statistics on the borrower's industry:

$$R_d = \frac{K}{L} \qquad [5]$$

- $\mathbf{R_d}$ is insurance margin;
- **K** is number of defaults in the borrower's industry for the period;
- **L** is number of loans extended by the bank to borrowers from the given industry for the period.

Or, there is a detailed alternative that gives consideration to lower significance of the technical default as compared to the complete one:

$$R_d = \frac{S_k}{S_a} \qquad [6]$$

- $\mathbf{R_d}$ is insurance margin;
- $\mathbf{S_k}$ is total arrears for the period;
- $\mathbf{S_a}$ is total of loans granted for the period.

We stress that amount of the insurance margin does not depend on inflation or other financial parameters, and is governed by only the absolute size of bankruptcies. This parameter is conservative and its value changes rather slowly.

Generally, this margin is indicative of that the bank sets up the insurance fund in addition to the borrower's security. In other words, this is an additional insurance mechanism that raises loan value and exists only in the high-risk economy.

2.3.2. Maximum business's rate (R_{max}) comprises the limit value of the borrowed funds cost which the business is prepared to pay. This limit value may be formalized as follows: from the business's viewpoint profit gained from the investment project depends on two entities including the business of which labor implements the project and the bank of which investments comprise a necessary condition for project implementation. Therefore, we may confirm that profit is a product made by two parties and the business will agree to distribute the profit on a 50% to 50% basis. Besides, the business's remaining funds must assure yield of at least mid-market level. Otherwise, the project may become pointless and unattractive for the business.

Now we shall formalize the aforesaid. We shall take EBIT, i.e. profit (PR) before interest (%) and tax (n) as a starting point:

$$EBIT = PR + \% + n$$

According to the condition, absolute profit and interest values are equal. To derive a formula according to the situation, we shall express profit using other parameters (EBIT, interest and tax) and similar designations X_{PR} and $X_{\%}$ for absolute profit and interest values, respectively:

$$EBIT = \frac{X_{PR}}{(1-n)} + X_{\%}$$

- $\mathbf{X_{PR}}$ is profit after tax;
- $\mathbf{X_{\%}}$ is total of expenses for interests on credits and loans;
- **n** is tax amount (income tax), in shares.

$$X_{PR} = (EBIT - X_{\%})(1-n)$$

Next, we shall develop equation:

$$X_{PR} = EBIT(1-n) - X_{\%}(1-n)$$

$$X_{PR} + X_{\%}(1-n) = EBIT(1-n)$$

Since absolute values of profit (X_{PR}) and interest ($X_{\%}$) are equal:

$$X + X - X n = EBIT(1-n)$$

$$X(1+1-n) = EBIT(1-n)$$

$$X = \frac{EBIT(1-n)}{(2-n)} \qquad [7]$$

So, X is the maximum (absolute) value of expenses for interest on a loan agreed to by the business. Since both EBIT and tax are charged on n annual basis, X value shall also be annual. Then maximum interest rate at which the business is prepared to take a loan will be equal to:

$$R_{\max} = \frac{X}{SK} \qquad [8]$$

- R_{max} is maximum interest rate at which the business is prepared to take a loan;
- **X** is maximum expenses for interests;
- **SK** is loan amount.

But since by X is also meant the income left with the business after maximum possible interest on a loan is paid, and X value is to be attractive for the business, then the following condition is to be met for the business:

$$R_{max} \geq R_c \qquad [9]$$

- R_{max} is maximum interest rate at which the business is prepared to take a loan;
- R_c is mid-market yield of alternative investments of the same risk category.

If the condition 9 is not met, then the condition for income distribution on a 50% to 50% basis is not possible and the procedure for calculation of maximum expenses (equation 7) is to be reviewed, i.e. maximum expenses for interest are to be calculated taking account of the profit left with business which is to be comparable to the mid-market yield of alternative projects. Hence, we shall again deal with the two parameters which now are not equal: X_{PR} и $X_{\%}$. This time maximum possible absolute value of expenses for interest shall be:

$$EBIT = \frac{X_{PR}}{(1-n)} + X_{\%}$$

$$X_{\%} = EBIT - \frac{X_{PR}}{(1-n)}$$

$$X_{\%} = EBIT - \frac{X_{PR}}{(1-n)}$$

Now we shall substitute X_{PR} parameter by the parameter derived from the formula 8 where we substitute R_c for R_{max}:

$$X_{\%} = EBIT - \frac{SK * R_c}{(1-n)} \qquad [10]$$

So “having a reserve from profit” sufficient to gain the mid-market yield, bank's shareholders shall be prepared to pay interest from the remaining EBIT. Then the maximum business's rate shall be derived from the following formula:

$$R_{\max} = \frac{X_{\%}}{SK} \quad [11]$$

Now, after the alternative-free situation (either to acquire a loan or abandon the project) has been reviewed and there is no opportunity to achieve the expected relationship between the bank's rate and bond yield, we shall proceed to the next chapter to review the situation when sources of financing are multiple.

3. Wide choice of alternatives: bank loan, own funds, bond issue and share issue

3.1. Project financing due to a bank loan

We shall proceed with interest rate analysis and review its relationship with parameters of the bond market which the closest to the banking sector out of all financial markets. To start, we shall weaken assumption No 2. Let me remind you, that assumption red as follows: a business has only one source of funds comprising bank loans and, hence, the business has only one alternative, i.e. either refuse the loan because too high interest rate or tale the loan and implement the project.

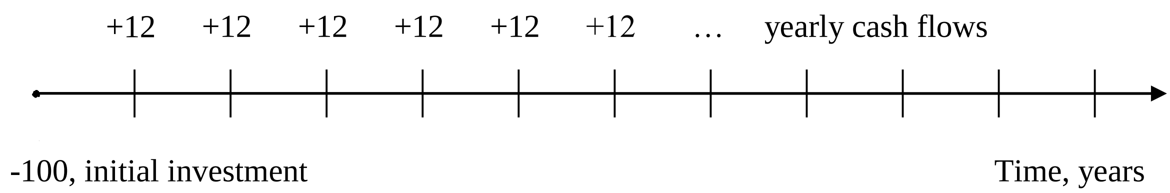
Now we shall reformulate assumption No 2 as follows: sources of project financing available for the business include: own funds, bank loans, issue of bonds and issue of shares.

So, the business has four alternative sources of project financing. Basically, the point is which of the sources of financing it will choose. It is evident that each source of financing influences the parameters of the investment project in its own manner: parameter may be better or worse. Since we proceed from the premise of rational business management (see assumption No 7), the main criterion of choice shall be the project net present value (NPV).

To make further analysis clear and understandable we shall illustrate it using a conventional project with following characteristics:

- investment size is 100 units;
- all investments are used concurrently in the starting period;
- yield anticipated by shareholders is 10% per annum;
- cash flow is12 units annually starting from the first year;
- cash flow is paid at the end of each year for unlimited number of years.

Pattern No 1 "Project cash flow":



The project has positive NPV:

$$NPV = -I + PV = -I + \frac{FC}{r} = -100 + \frac{12}{0.1} = -100 + 120 = +20$$

- **NPV** is project net present value;
- **I** is amount of project investments (raised loan);
- **PV** is project cash flow present value;
- **FC** is yearly project cash flow.

This project has been developed in view of financing with a bank loan and, hence, time and size of cash flows imply relatively prompt acquiring of funds (loan) and their rapid use.

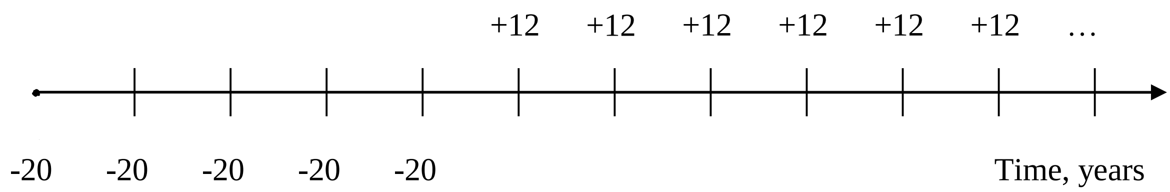
Now refusal to raise a bank loan and substitution of it by another source of financing shall result in the following:

- first, changes of project implementation deadline;
- second, changes in cash flows due to change in cost of raised funds.

Let us consider how changes of the first factor, i.e. time of financing, shall influence the project (the second factor shall be added to the analysis later). To do so, we shall first consider financing using own funds and then due to issue of bonds, and at last financing due to issue of shares.

3.2. Project financing using own funds

Let us suppose that financing of the project due to own funds is possible though it makes implementation time longer. For instance, the business may annually invest 20 units of own funds. Then the project is extended for five years, and the cash flow pattern shall look like as follows (pattern No 2):



Change in time results in change in NPV which becomes here negative:

$$NPV = -I + PV = -20 - \frac{20}{(1+r)} - \frac{20}{(1+r)^2} - \frac{20}{(1+r)^3} - \frac{20}{(1+r)^4} + \frac{1}{(1+r)^4} * \frac{12}{r} =$$

$$NPV = -20 - \frac{20}{1.1} - \frac{20}{1.21} - \frac{20}{1.33} - \frac{20}{1.46} + \frac{1}{1.46} * \frac{12}{0.1} =$$

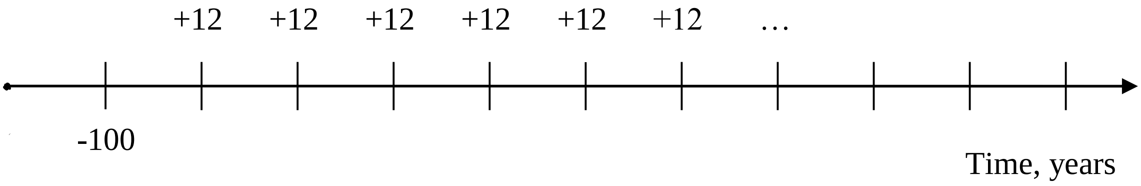
$$NPV = -20 - 18.18 - 16.53 - 15.04 - 13.69 + 82.19 =$$

$$NPV = -83.44 + 82.19 = -1.25$$

So, use of own funds for project financing makes it unprofitable.

3.3. Project financing due to bond issue

Apparently, refusal to raise a bank loan and use of bond issue instead of it will also influence the time of project implementation: it will start one year later (assumption No 8). The above one-year period is chosen because this is the minimum necessary time to prepare documents and take all measures required within the frame of the bond issue. This time period is sufficient for any issuer. Then the cash flow pattern shall look like as follows (pattern No 3):



That is, everything shall occur one year later and NPV shall be defined as follows:

$$NPV = -I + PV = -\frac{100}{(1+r)} + \frac{1}{(1+r)} * \frac{12}{r} =$$

$$NPV = -\frac{100}{1.1} + \frac{1}{1.1} * \frac{12}{0.1} =$$

$$NPV = -\frac{100}{1.1} + \frac{120}{1.1} = -90.90 + 109.09 = +18.19$$

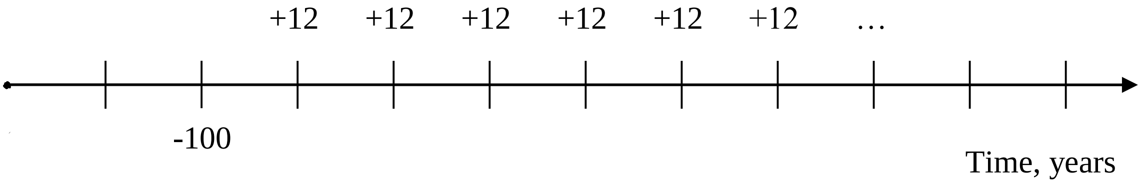
However, the result is to be corrected in view of issue costs. As maximum, they may amount to 5% of the issue size. Hence, in order to obtain a correct result, the absolute investment amount is to be incremented by 5%. Then, we shall obtain:

$$NPV = -\frac{105}{1.1} + \frac{120}{1.1} = -95.45 + 109.09 = +13.64$$

So, the project has positive NPV and may be implemented.

3.4. Project financing due to share issue

Share issue as one of the ways to attract investments shall also cause change in the start-up deadline. As in the case with bond issue, the chosen deadline depends on size and duration of preliminary operations and the procedure of initial public offering (IPO). In the case we deal with the time period shall reach two years (assumption No 9). This time period is sufficient for most of issuers. Then the cash flow pattern shall look like as follows (pattern No 4):



Cash flow pattern shall occur two years later and project NPV shall look like as follows:

$$NPV = -I + PV = -\frac{100}{(1+r)^2} + \frac{1}{(1+r)^2} * \frac{12}{r} =$$

$$NPV = -\frac{100}{1.21} + \frac{1}{1.21} * \frac{12}{0.1} =$$

$$NPV = -\frac{100}{1.21} + \frac{120}{1.21} = -82.64 + 99.17 = +16.53$$

But the result is to be corrected in view of size of IPO expenses. At worst it may amount to 10% of the issue size. Hence, in order to obtain a correct result, the absolute investment amount is to be incremented by 10%. Then, one obtains:

$$NPV = -\frac{110}{1.21} + \frac{120}{1.21} = -90.91 + 99.17 = +8.26$$

NPV is above zero and the project may be implemented.

3.5. Choice of the source of financing

Now, before proceeding to analysis of the second factor comprising changes in cash flow size due to changes in the cost of raised funds, we shall tabulate all the data obtained. See table 1.

The table clearly illustrates differences between various ways of financing including those against the final output, i.e. net present cost. However, the result obtained may appreciably change if we give consideration to the second factor comprising various costs of raised funds.

Since our project was simplified (including the condition that we neglect the cash flow structure) from the very beginning, its use is not reasonable any more. Instead, however, we may state which is apparent that cost of funds (when bond issue, own funds and share issue are used) shall appreciably lower than on the case of bank loan. In addition, when the two last ways of raising funds are used, the factor of interest payment before tax is omitted. As a result, true NPV as to these three ways shall be appreciably higher than in the table 1.

No	Sources of financing	First factor*				Second factor *	NPV (exclusive of call. 5)
		Change of project starting date	Extension of project deadline	Discounted rate for extension period	Borrower's expenses for raising funds**	Cost of funds	
A	B	1	2	3	4	5	6
1.	Bank loan	-	-	-	-	P_{cr}	+20
2.	Own funds	-	5 years	r	-	P_d	-1.25
3.	Bond issue	1 year	-	r	5	P_b	+13.64
4.	Share issue	2 years	-	r	10	P_d	+8.26

Table 1. Project net present value with various sources of financing

* is factors affecting NPV due to substitution of bank loan by another source of financing;

** is in percents from the amount raised;

$\mathbf{P_{cr}}$ is bank's loan interest rate;

$\mathbf{P_d}$ is dividend yield;

P_b is yield to bond redemption.
r is yield anticipated by shareholders.

However, using the logic of competition for customers if a source of financing reduces the cost of funds offered to the borrower, it will bring the project NPV to the level when it becomes attractive to the maximum.

In addition, if NPV in two any different alternatives of financing are equal, it means that the difference between costs of funds in two different alternatives shall be the largest. For instance, if project NPV in case of bank loan is equal to project NPV in case of bond issue, it means that the difference between the bank's interest rate and bond yield shall be the largest.

Strictly speaking, under conditions of strong competition for a customer (borrower) arbitration is to make difference between NPV in different project financing alternatives the lowest. It means that:

$$NPV_{Bank} = NPV_{Bond} = NPV_{Stock} = NPV_{Own} \qquad [12]$$

Then, if to review the assumption No 10 – on financial markets competition between suppliers of financial resources for customers is strong – we may find the relationship between the bank's interest rate and yield on the bond market. To do so, we, first, shall consider the relationship between the project cash flows calculated using two different sources of financing and then based on this relationship we shall consider the relations between the bank's loan rate and bond yield.

3.6. Cash flows as a main summand of net present value

Now we shall consider a particular case of the above equation (see formula 12):

$$NPV_{Bank} = NPV_{Bond}$$

We shall describe each of the parameters:

$$-I + \frac{FC_{Bank}}{r} = -I * \frac{1}{1+r} + \frac{FC_{Bond}}{r} * \frac{1}{1+r} \qquad [13]$$

- **NPV_{Bank}** is project net present value when a source of financing comprises a bank loan;
- **NPV_{Bond}** is project net present value when a source of financing comprises bond issue;
- **FC_{Bank}** is project resultant cash flow when a source of financing comprises a bank loan, annually;
- **FC_{Bond}** is project resultant cash flow when a source of financing comprises share issue, annually;
- **I** is amount of project investments (amount of a raised loan);
- **r** is yield anticipated by business's shareholders (project investors).

$$\frac{FC_{Bank}}{r} = I - I * \frac{1}{1+r} + \frac{FC_{Bond}}{r} * \frac{1}{1+r}$$

$$\frac{FC_{Bank}}{r} = \frac{I(1+r)}{1+r} - I * \frac{1}{1+r} + \frac{FC_{Bond}}{r} * \frac{1}{1+r}$$

$$\frac{FC_{Bank}}{r} = \frac{I(1+r) - I +}{1+r} + \frac{FC_{Bond}}{r} * \frac{1}{1+r}$$

$$FC_{Bank} = r * \left(\frac{I * r}{1+r} + \frac{FC_{Bond}}{r} * \frac{1}{1+r} \right)$$

$$FC_{Bank} = \frac{I * r^2 + FC_{Bond}}{1+r} \qquad [14]$$

Next step shall consist in describing FC (cash flow) parameter and deriving the relationship between the bank interest rate and bond yield.

3.7. Cost of funds as a component of cash flow

As is known, cash flow includes two key summands: depreciation and profit after tax. Therefore we have to express cash flow so that the formula contains an interest rate. To do so, we shall express profit using EBIT (this parameter is more accessible than EBITDA). It should be noted that EBIt will be the same in both variants of cash flow calculations. Then we shall derive bank loan interest rate and bond yield from profit:

$$Profit = EBIT - interest - tax$$

$$FC = Depr + E - C * N - (E - C * N) * n$$

- **FC** is cash flow;
- **Depr** is depreciation;
- **E** is EBIT, i.e. project profit before interest and tax;
- **C** is debt interest rate;
- **N** is nominal debt amount;
- **n** is income tax rate for legal entities.

Then:

$$FC = Depr + (E - C * N) * (1 - n) \qquad [15]$$

Now we shall substitute both cash flow parameters in equation 14 using formula 15:

$$FC_{Bank} = \frac{I * r^2 + FC_{Bond}}{1+r}$$

$$Depr + (E - C_{Bank} * N) * (1 - n) = \frac{I * r^2 + Depr + (E - C_{Bond} * N) * (1 - n)}{1+r}$$

$$Depr*(1+r)+(E-C_{Bank}*N)*(1-n)*(1+r)=I*r^2+Depr+(E-C_{Bond}*N)*(1-n)$$

$$Depr*r+(E-C_{Bank}*N)*(1-n)*(1+r)=I*r^2+(E-C_{Bond}*N)*(1-n)$$

$$(E-C_{Bank}*N)*(1-n)*(1+r)-(E-C_{Bond}*N)*(1-n)=I*r^2-Depr*r$$

$$(1-n)*((E-C_{Bank}*N)*(1+r)-(E-C_{Bond}*N))=I*r^2-Depr*r$$

$$(E-C_{Bank}*N)*(1+r)-(E-C_{Bond}*N)=\frac{I*r^2-Depr*r}{(1-n)}$$

$$E-C_{Bank}*N+r*E-r*C_{Bank}*N-E+C_{Bond}*N=\frac{I*r^2-Depr*r}{(1-n)}$$

$$r*E-C_{Bank}*N-r*C_{Bank}*N+C_{Bond}*N=\frac{I*r^2-Depr*r}{(1-n)}$$

$$r*E-C_{Bank}*(N-r*N)+C_{Bond}*N=\frac{I*r^2-Depr*r}{(1-n)}$$

$$-C_{Bank}*(N-r*N)=\frac{I*r^2-Depr*r}{(1-n)}-r*E-C_{Bond}*N$$

$$C_{Bank}*(N-r*N)=C_{Bond}*N+r*E-\frac{I*r^2-Depr*r}{(1-n)}$$

$$C_{Bank}=\frac{C_{Bond}*N}{N-r*N}+\frac{r*E}{N-r*N}-\frac{I*r^2-Depr*r}{(1-n)*(N-r*N)}$$

$$C_{Bank}=\frac{C_{Bond}*N}{(1-r)*N}+\frac{r*E}{(1-r)*N}-\frac{I*r^2-Depr*r}{(1-n)*(1-r)*N}$$

$$C_{Bank}=\frac{C_{Bond}}{(1-r)}+\frac{E*r}{(1-r)*N}-\frac{(I*r-Depr)*r}{(1-n)*(1-r)*N}$$

$$C_{Bank}=\frac{1}{(1-r)}*\left(C_{Bond}+\frac{E*r}{N}-\frac{(I*r-Depr)*r}{(1-n)*N}\right)$$

$$C_{Bank}=\frac{1}{(1-r)}*\left(C_{Bond}+\frac{r}{N}*\left(E-\frac{I*r-Depr}{(1-n)}\right)\right) \quad [16]$$

Two parameters in formula No 16, i.e. **N** and **I,** mean the same. **N** is the total of raised funds, and **I** is the amount of project investments. Substantially, this is the same with the proviso that in the case of bond issue loan amount is to be a little bit higher (a few percents) than the amount needed for project financing. This is due to expenses for issue operations. However, this difference is insignificant (mostly within five percents) and, therefore, it may be neglected. Hence, **N** = **I**.

Then the basic formula of the author's bank interest rate model shall be as follows:

$$C_{Bank} = \frac{1}{(1-r)} * \left(\left(E - \frac{I * r \quad - Depr}{(1-n)} \right) * \frac{r}{I} + C_{Bond} \right) \qquad [17]$$

or:

$$C_{Bond} = C_{Bank} * (1-r) - \frac{r}{I} * \left(E - \frac{I * r \quad - Depr}{(1-n)} \right) \qquad [18]$$

The last step to make is to explain the procedure for calculations of bond yield, $\mathbf{C_{bond}}$. We would like to remind that this is yield of bonds to be placed in a year.

In the course of our analysis we neglected the credit (loan) term because this parameter was not directly used in the calculations. However, now we have to apply it, since the only way to ensure a formalized approach to future bond yield is to use the pure expectations theory (the model of temporary interest rate structure).

According to this theory, if an average term for placement of bonds (we would like to remind once again that placement shall occur in a year) is, for instance, one year, then for calculations of the forward yield rate we have to know current yield of a one-year and two-year bond. In the case of two years – yield of a two-year and tree-year bond. In the case of four years – yield of a tree-year and four-year bond etc.

For instance, for a one-year case formula No 17 shall be written as follows:

$$C_{Bank} = \frac{1}{(1-r)} * \left(\left(E - \frac{I * r \quad - Depr}{(1-n)} \right) * \frac{r}{I} + \frac{C^2_{2years}}{(1+C_{1year})} - 1 \right) \qquad [19]$$

- $\mathbf{C_{2years}}$ is current yield of a two-year bond;
- $\mathbf{C_{1year}}$ is current yield of a one-year bond.

For simplicity and usability formulae 17 and 18 may be rewritten as follows:

$$C_{Bank} = \frac{k + C_{Bond}}{(1-r)} \qquad [20]$$

or:

$$C_{Bond} = C_{Bank} * (1-r) - k \qquad [21]$$

where **k** factor is equal to:

$$k = \left(E - \frac{I * r \quad - Depr}{(1-n)} \right) * \frac{r}{I} \qquad [22]$$

Now, after the paper objective has been achieved, we shall consider characteristics of the resultant bank interest rate model in the next chapter.

4. Graphical analysis of the bank interest rate model

4.1. Bank interest rate as a function of bond yield

Summarizing provisions of the fist and second chapters we approach the description of the bank's interest rate model: **"Bank's interest rate lies in the range defined by formulae No 1 and No 8, while its calculated value may be defined by the formula No 17"**.

Now, we shall review application of the resultant model (formula No 17) using a simple numerical example which makes it possible to analyze variability of the bank's interest rate:

- Yield anticipated by shareholders – 30%
- Project investments – 100 units
- Depreciation – 20 unit/year
- EBIT – 30 unit/year
- Income tax – 24%

Fig. 1 shows calculated bank's interest rate versus the main variable, i.e. forward bond yield. For instance, at future 10% yield on the bond market, the current bank's interest rate is to be 21.5%.

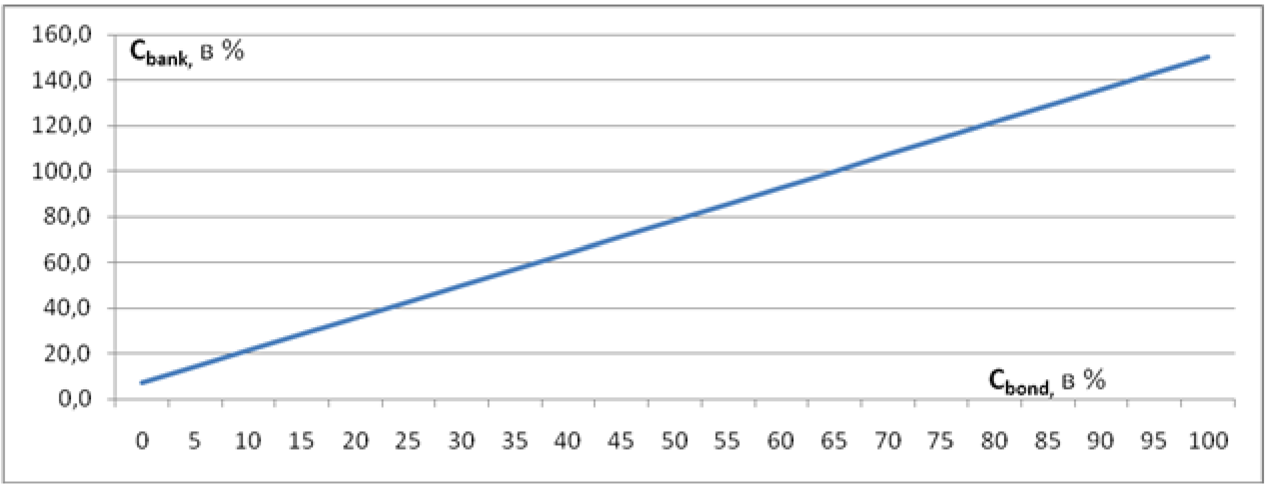


Fig. 1. Curve of current interest rate on bank loans versus future yield on the bond market (We would like to remind that r = 30%)

Relationship between both parameters is linear: increment of one parameter causes increment of the other one, and vice versa. In this case, the formula does not imply that any of these parameters is leading and the second one is led. On markets where the stock market and banking sector are appreciably developed, the both parameters are interdependent. However, if one of the markets is weak, influence shall be one-sided.

Change in parameters of the formula influences the relationship curve in two ways. First, when the parameters characterizing the project success (investments, depreciation, EBIT and income tax) change, the relationship curve shall shift symmetrically upward on improvement or downward on aggravation. In this case the inclination angle shall remain unchanged. This is explained by that on improvement of the financial project indicators the size of resources that may be used for paying capital grows.

So, the lower the quality of investment projects all over the country, the less the difference in absolute terms between the bank's rate and bond yield, and vice versa, the higher the quality of investment projects, the larger the above difference.

Second, when the "anticipated yield" parameter changes, the relationship curve inclination angle also changes: when the indicator size grows, the angle increases, and vice versa, when the anticipated yield decreases, the angle also decreases. See Fig. 2 where the "anticipated yield" parameter has grown from 30% to 70% as compared to Fig. 1. As a result, the inclination angle has increased and the curve has shifted rightward which means that under these conditions the bond yield below 25% is impossible even theoretically.

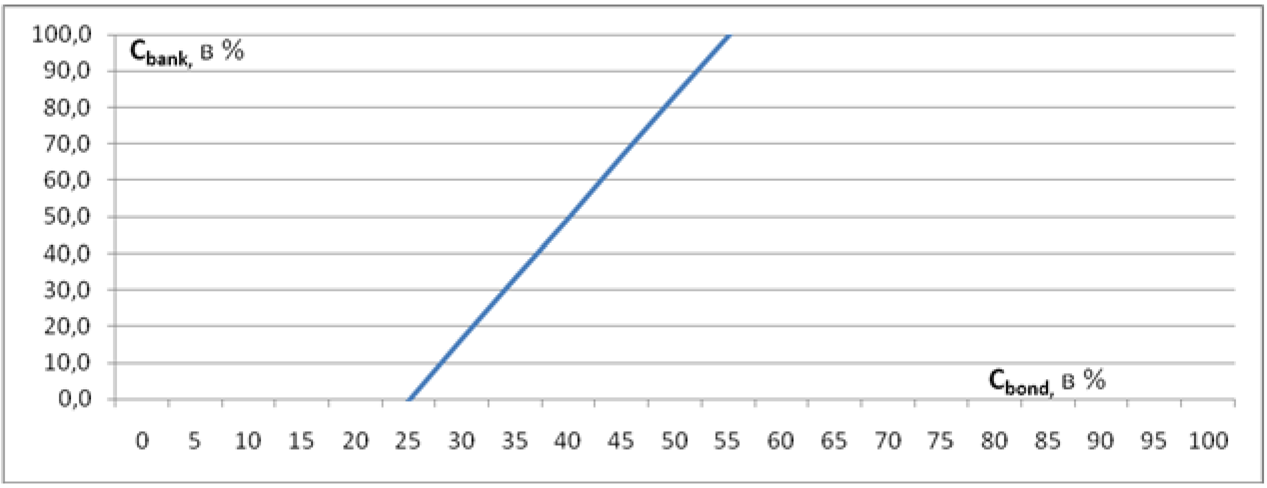


Fig. 2. Curve of current interest rate on bank loans versus future yield on the bond market (at r = 70%)

At zero anticipated yield the bank's rate and bond yield shall change absolutely on a pro rata basis (the bank's rate shall change by one percent per each percent in change of the bond yield rate). In this case the relationship curve inclination angle shall reach minimum 45 degrees. See Fig. 3.

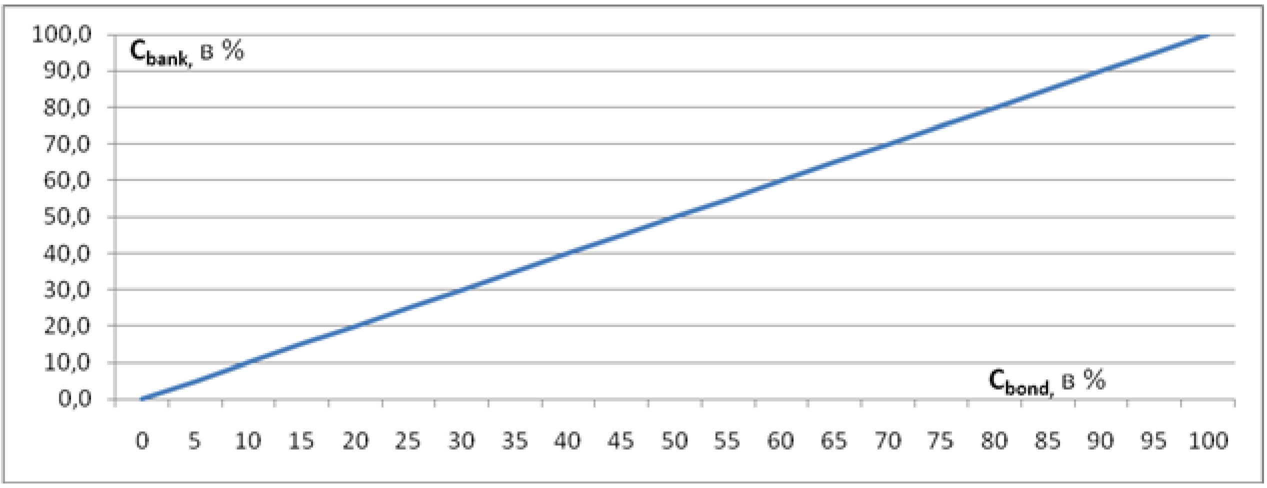


Fig. 3. Curve of current interest rate on bank loans versus future yield on the bond market (at r = 0%)

At 100% anticipated yield the model may not be applied since in one of the formula components the need for division by zero occurs.

At nearly 100% anticipated yield the relationship curve inclination angle is 90 degrees. See Fig. 4. Under these conditions the bank's interest rate shall not depend on the bond market yield and yield shall have the only value close to 75% (we would like to remind that these are calculations at predetermined project indicators).

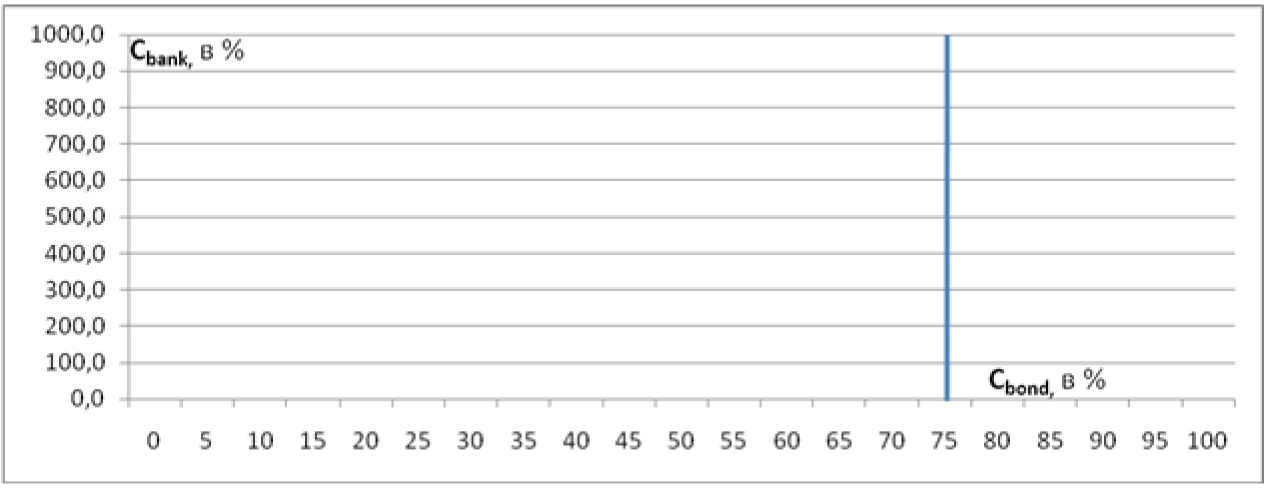


Fig. 4. Curve of current interest rate on bank loans versus future yield on the bond market (at r ≈ 100%)

When the anticipated yield size is incremented up to the value of 120, the relationship curve inclination angle changes. See Fig. 5. Now the curve goes from left to right, from top downward.

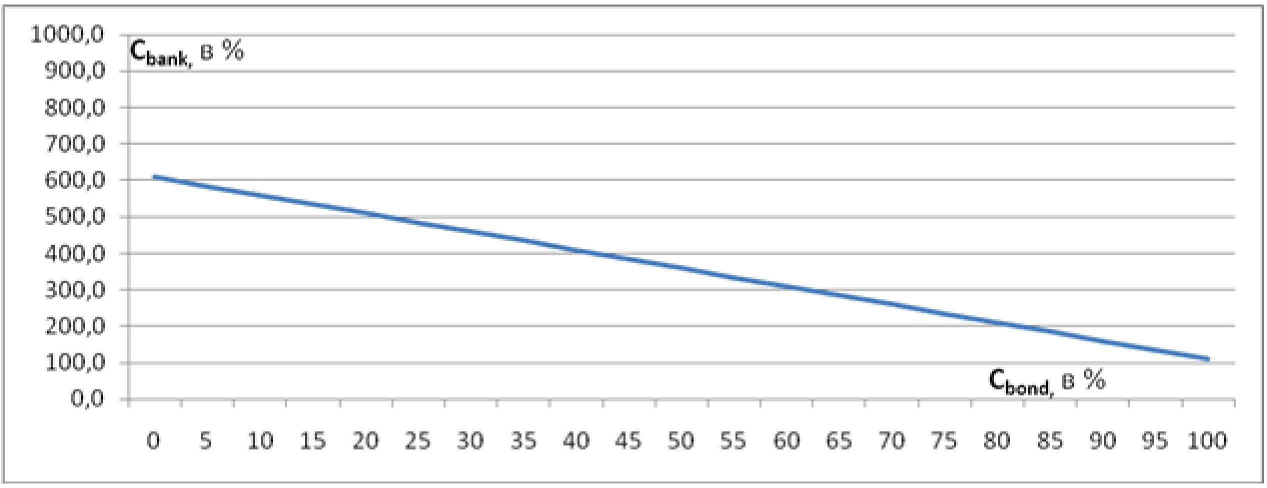


Fig. 5. Curve of current interest rate on bank loans versus future yield on the bond market (at r = 120%)

If the anticipated yield size is incremented on, the curve gradually takes strictly horizontal position and shows indifference of the bank's interest rate to the bond market. See Fig. 6.

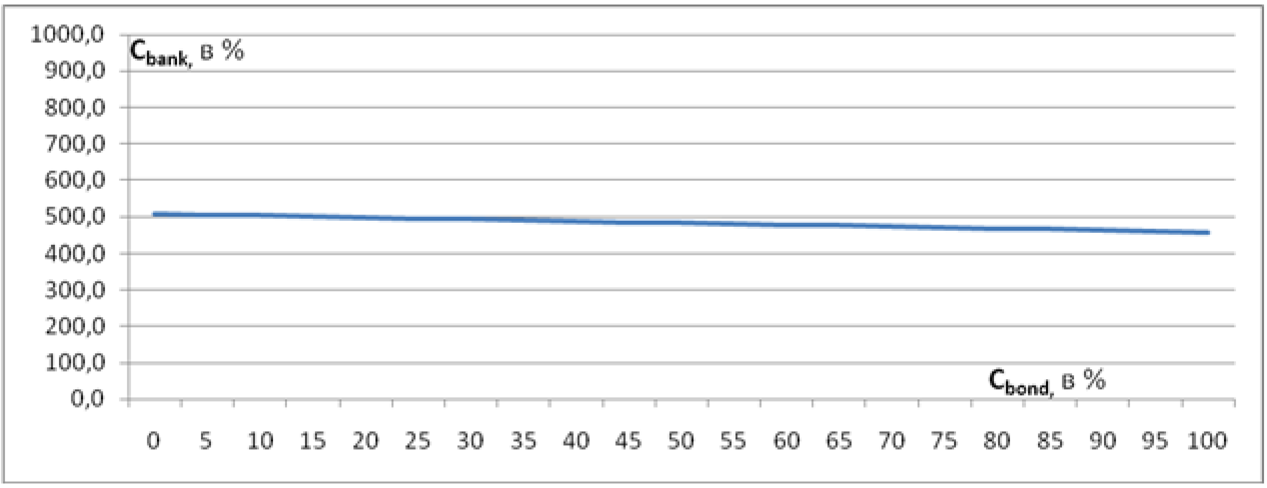


Fig. 6. Curve of current interest rate on bank loans versus future yield on the bond market (at r = 300%)

As a whole, the following observation may be formulated: the relationship curve inclination angle may play a role of the indicator comprehensively reflecting the situation in the finance sector: the closer the inclination angle to 45 degrees, the better situation in the finance sector.

4.2. Bank's rate as a function of the yield anticipated by shareholders

In view of strong influence of the anticipated yield on the relationship curve of both rates, dependence of the bank's interest rate on the anticipated yield is of interest. Fig. 7 shows the results of such calculations: size of the bank's interest rate as a function of variations of the anticipated yield (the preset bond yield is 8%).

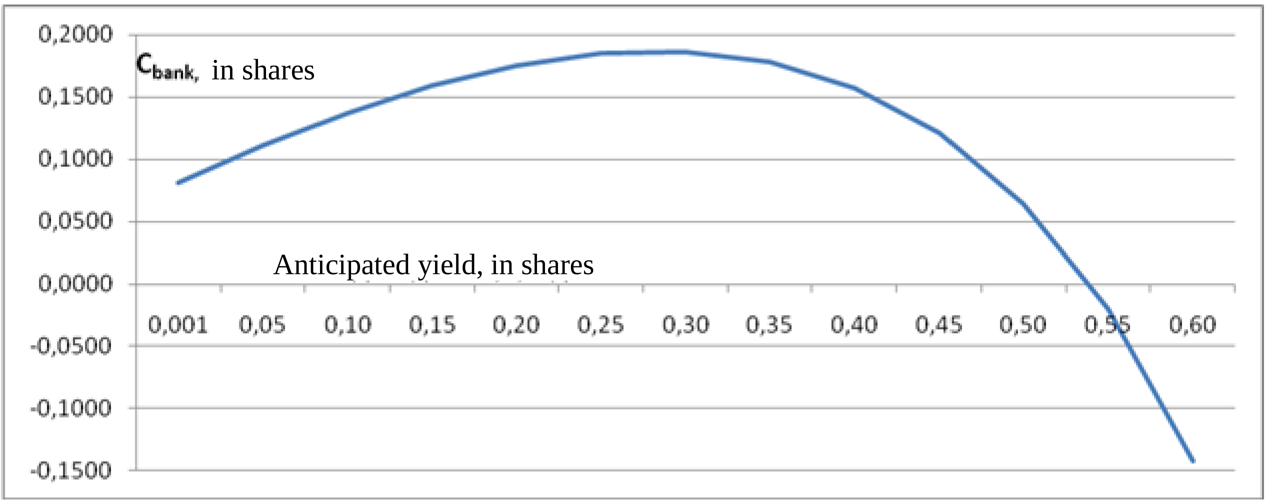


Fig. 7. Curve of interest rate on bank loans versus yield anticipated by shareholders (at C_{bond} = 8%)

The curve of relationship between the bank's rate and the anticipated yield comprises an arc. For instance, 15% bank's rate simultaneously corresponds to two values of the anticipated yield of 12% and 42%. In other words, if we know values of the bank's rate and bond yield and try to calculate the value of the anticipated yield, we shall obtain two results. Such phenomenon may be explained by payment of interest on debt before tax.

As the values grow (the left part of the curve) interests of the bank and lender shall coincide and they both shall strive to increase the values (of loan rate and anticipated yield, respectively). But as the right part of the curve shows, after the peak value interests of the parties are diametrically opposite: the borrower shall strive to increment the anticipated yield, while the bank will stand against it in every possible way.

There is the only value of the anticipated yield when the bank loan rate is of maximum value. Bearing in mind that banking sector as any supplier of services always seeks to maximize the loan rate, the given value of the anticipated yield (see Fig. 7 for the points with coordinates: 30% anticipated yield and 18.6% bank's interest rate) shall be the average anticipated yield which investors are to be practically guided by.

At a specified, rather high value of the anticipated yield (r = 55% in Fig. 7) the loan spot rate shall be amount to zero. It means that such value of the anticipated yield may be reached only in case of bank loan at zero interest rate. In addition, this is a border line. If it is crossed, the given model may not be applied.

Speaking of the relationship curve one may additionally stress that with growing bond yield the curve rises, and with dropping yield the curve goes down. When the curve goes up, the direction of the right part thereof changes: if at relatively low bond yield values it hangs down, at high values it picks (see Fig. 8 and 9).

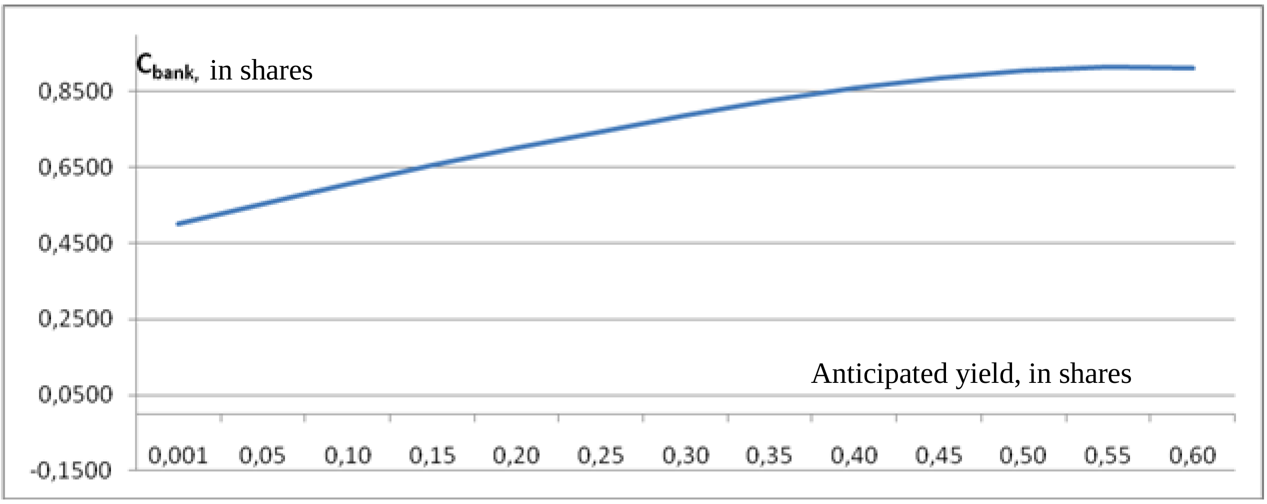


Fig. 8. Curve of interest rate on bank loans versus yield anticipated by shareholders (at C_{bond} = 50%)

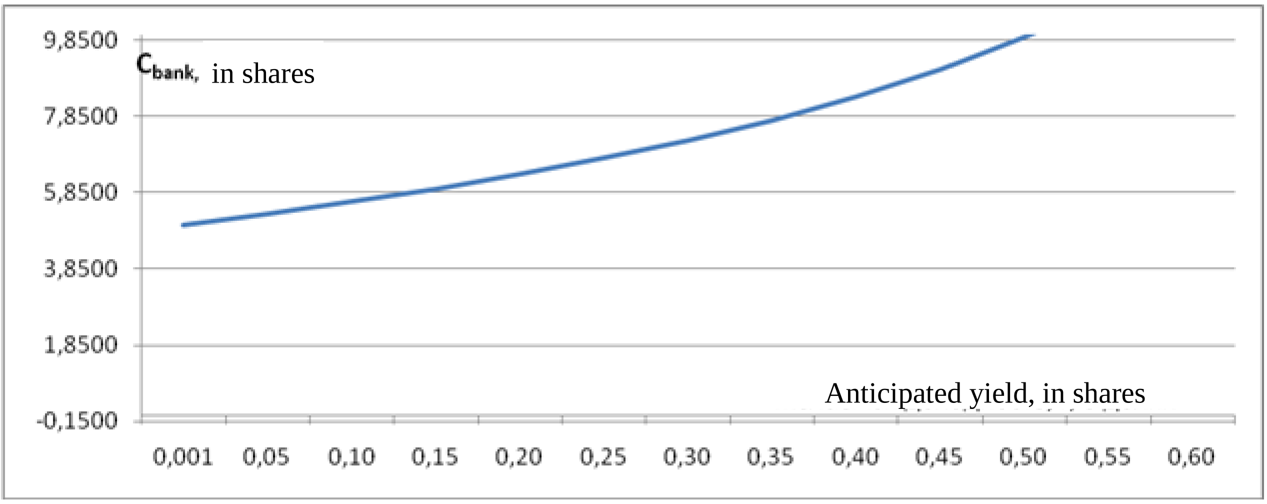


Fig. 9. Curve of interest rate on bank loans versus yield anticipated by shareholders (at C_{bond} = 500%)

Under hyperinflation conditions at bond yield rate of hundreds of percents the interests of the parties coincide over the whole curve.

5. Practical application of bank's interest rate model

5.1. Calculation procedure

Calculations using formula 17 have been made in early 2010 employing materials of the Russian businesses. Since this formula contains a one-year time lag between the bank loan rate and bond yield, there is an opportunity to define the future corporate bond yield rate on the basis of current financial indicators and, hence, appraise development of the Russian economy in the near future.

This objective has been achieved in three stages. First, we selected issuing companies of which materials we used for calculations. Second, we calculated values of the yield anticipated by shareholders in 2007, 2008, and 2009. Third, we calculated corporate bond yield in early 2011.

First stage. At the beginning, all Russian businesses that issued corporate bonds, some six hundred in early 2010, were divided by their industry affiliation. Then we chose 6 most representative industries for analysis with 5 issuing companies in each industry (selection criteria included company size, issue volume, rating). Totally we chose 30 companies, though to the end of calculations we dropped two companies due to lack of data.

Stage 2. We calculated yield anticipated by shareholders for each company in 2007, 2008 and 2009. Then we predicted the yield the shareholders are willing to obtain in 2010 for each company using an expert approach.

Stage 3. Based on extrapolation of data in financial reports in 2009 (at the time of calculations we had reports for 2nd and 3rd quarters of 2009 rather than for the whole year of 2009, therefore, we made calculations using data in the financial reports to complete the year) and the yield anticipated by shareholders predicted at the second stage we made calculations of the yield of bonds expected in early 2001.

We have to state that we had no information about the interest rate the issuing companies raised commercial loans, therefore we used the average bank loan rate of the Central Bank of Russia for all companies.

To practically apply formula 17 we had to specify one aspect related to Investments indicator. According to the logic of this formula, this indicator is taken to mean the sum of investments into the future investment project. But since we undertook calculations in relation to long operating companies rather than new projects, instead of the Investments indicator we shall use Company Equity indicator (the sum of shareholders' capital and borrowed capital of a company as of the end of a year) as the closest in essence to the formula 17.

5.2. Results of Calculations

Based on the calculation made (see Annex 1) we derived the following observation: to early 2011 the yield of bonds of gas and oil companies shall drop and that of the companies in other industries shall rise. It may be explained that the financial position of companies in the raw materials industry shall improve and that of in other sectors it will worsen.

Also, in 2008 the yield anticipated by shareholders in many companies has decremented which may be explained by preparedness of shareholders to save in the financial recession.

In most of cases calculations of the yield anticipated by shareholders produced a correct and probable result. Therefore, we may assert that formula 17 may be practically applied.

However, in a number of cases (to be exact, in 11 out of 27) we received incorrect values of indicators (which are replaced by dashes in the tables) which may be explained by incorrect financial reports published by issuing companies. In addition, we stress that when calculations produce non-realistic low values of the yield anticipated by shareholders, it may mean that for the shareholders the dividends are not the main source of income and their size are not of critical importance. The worst situation was observed in the civil engineering industry (i.e., calculations in relation to all companies produced incorrect values). A little bit better it was observed in food industry. Hence, on reviewing financial reports of companies in these two industries, one should be particularly careful.

6. Conclusions

6.1. There are two borderline values of interest rate within which one defines the real rate: the minimum rate at which the bank lends a loan and the maximum one at which a business may raise a loan.

6.2. Governmental policy in monetary regulation is to be oriented to a wider range between the minimum and maximum rates rather than reduction of bank loan rate.

6.3. Relationship between the bank's interest rate and bond yield is expressed by the following formula:

$$C_{Bank} = \frac{k + C_{Bond}}{(1-r)}$$

- $\mathbf{C_{Bank}}$ –is bank interest rate
- $\mathbf{C_{Bond}}$ is bond yield
- **r** is yield anticipated by shareholders
- **k** is ad hoc factor

k factor is calculated as follows:

$$k = \left(E - \frac{I * r \ - Depr}{(1-n)} \right) * \frac{r}{I}$$

- **Depr** is depreciation;
- **E** is EBIT, project profit before interest and tax;
- **r** is yield anticipated by shareholders;
- **I** is investments (business's equity);
- **n** is income tax rate for legal entities.

6.4. Relationship between $\mathbf{C_{Bank}}$ and $\mathbf{C_{Bond}}$ is not permanent. To a great extent it depends on the size of the yield anticipated by shareholders. When the anticipated yield lies within the range from zero to 100%, the relationship between the bank's rate and bond yield is directly proportional. At the anticipated yield of 100% the model is not applicable. At the anticipated yield above 100% the relationship is inversely proportional.

6.5. The inclination angle of the curve of relationship between the current bank loan interest rate and future yield on the bond market may play a role of the indicator comprehensively reflecting the current situation in the finance sector: the closer the inclination angle to 45 degrees, the better situation in the finance sector.

7. Annex

Calculated corporate bond yield in Russia as of early 2011

The calculations have been carried out on the basis of data from the official site of the Central Bank of Russia (http://www.cbr.ru), ad hoc site http://www.rusbond.ru/, and official sites of the companies issuing bonds.

7.1. Oil and Gas Industry

(calculations have been carried out by Irina Kelbrant, Kazakhstan Branch of Lomonosov MSU)

Yield to bond redemption

	Actual values*, %			Calculated values*, %
	2008	**2009**	**2010**	**2011**
NK Alliance	10.14	9.87	13.45	10.01
Gazprom	9.76	10.32	13.23	11.34
Integra Finance	7.35	8.57	12.80	13.61
Lukoil	7.57	9.43	13.57	10.54
Rusneft	6.85	7.26	11.64	10.04

* as of the beginning of the year

Calculated yield anticipated by shareholders

	Calculated values *, %			Predicted values**, **2010, %**
	2007	**2008**	**2009**	
NK Alliance	3.44	8.59	13.35	11.45
Gazprom	6.88	4.3	14.51	11.00
Integra Finance	31.21	21.17	17.71	11.03
Lukoil	27.91	12.63	12.38	11.01
Rusneft	36.10	33.84	25.41	10.87

* obtained from formula 17

** Determined by experts on the basis of kinetic profile of values in 2007-2009

7.2. Mechanical Engineering

(calculations have been carried out by Xenia Kornienko, Kazakhstan Branch of Lomonosov MSU)

Yield to bond redemption

	Actual values*, %			Calculated values*, %
	2008	**2009**	**2010**	**2011**
GAZ-Finance	14.94	32.23	27.14	30.14
MiG-Finance	2.03	15.56	14.43	17.40
OMZ-5	11.18	14.32	13.39	16.39
NPO Saturn	11.98	9.74	36.50	39.50
Transmash Holding	9.30	39.58	6.51	46.51

* as of the beginning of the year

Calculated yield anticipated by shareholders

	Calculated values *, %			Predicted values**, **2010, %**
	2007	**2008**	**2009**	
GAZ-Finance	27.1	53.7	50.8	50.8
MiG-Finance	–	35.4	26.3	26.3
OMZ-5	14.6	26.9	25.5	25.5
NPO Saturn	–	29.9	53.3	53.3
Transmash Holding	–	59.9	–	59.9

* obtained from formula 17

** Determined by experts on the basis of kinetic profile of values in 2007-2009

7.3. Food Industry

(calculations have been carried out by Maxim Korniaev, Izhevsk State Technological University)

Yield to bond redemption

	Actual values*, %			Calculated values*, % **2011**
	2008	**2009**	**2010**	
Agrocom Group	10.14	9.87	13.45	39.53
JFC Group	9.67	10.32	13.23	–
Cherkizovo Group	7.35	8.57	12.80	–
PAVA	7.57	9.43	13.57	–
Unimilk Finance	6.85	7.26	11.64	–

* as of the beginning of the year

Calculated yield anticipated by shareholders

	Calculated values *, %			Predicted values**, **2010, %**
	2007	**2008**	**2009**	
Agrocom Group	3.39	8.69	13.37	13.37
JFC Group	–	–	0.01	
Cherkizovo Group	12.33	9.88	7.35	7.35
PAVA	–	–	–	
Unimilk Finance	–	–	–	

* obtained from formula 17

** Determined by experts on the basis of kinetic profile of values in 2007-2009

«–» incorrect results

7.4. Power Engineering

(calculations have been carried out by Nairi Markarian, Lomonosov MSU)

Yield to bond redemption

	Actual values*, %			Calculated values*, % **2011**
	2008	**2009**	**2010**	
Mosenergo	7.6	9.78	8.65	–

UES FSK	7.8	9.30	7.85	–
OGK-3	7.46	10.20	8.41	–

* as of the beginning of the year

Calculated yield anticipated by shareholders

	Calculated values *, %			Predicted values**, **2010, %**
	2007	**2008**	**2009**	
Mosenergo	51.15	53.3	39.29	34.43
UES FSK	50.4	30.25	33.7	27.55
OGK-3	34.8	90.7	38.76	40.90

* obtained from formula 17

** Determined by experts on the basis of kinetic profile of values in 2007-2009

7.5. Civil Engineering

(calculations have been carried out by Dana Kupenova, Kazakhstan Branch of Lomonosov MSU)

Yield to bond redemption

	Actual values*, %			Calculated values*, % **2011**
	2008	**2009**	**2010**	
Glavstroi Finance	19.68	7.50	15.75	–
Cosmos Finance	17.20	17.20	21.10	–
LSR-Invest	27.76	25.83	12.84	–
LEK-Stroi	0.1	0.1	0.1	–

* as of the beginning of the year

Calculated yield anticipated by shareholders

	Calculated values *, %			Predicted values**, **2010, %**
	2007	**2008**	**2009**	
Glavstroi Finance	–	–	–	
Cosmos Finance	–	–	–	
LSR-Invest	–	–	–	
LEK-Stroi	–	–	–	

* obtained from formula 17

** Determined by experts on the basis of kinetic profile of values in 2007-2009

«–» incorrect results

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