Regret Aversion and Demand for Mixed Insurance

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1. Introduction

"Insurance" in Oxford Dictionary of English

an arrangement by which a company of the state undertakes to provide a guarantee of compensation for specified loss, damage, illness, or death in return for payment of a specified premium

We call this type of insurance "**term insurance**".

This explanation seems to be reasonable. But, in Japanese insurance market, other type of insurance called "**mixed insurance**" is widely distributed. By holding mixed insurance, insured does not only receive payment in loss realization, but also payment when she does not suffer any loss.

- This type of insurance accounts for about 15 percentage of Japanese insurance market. This can not be negligible since Japanese insurance market is huge. (The amount of new polices including conversion is 65.60 trillion yen! 656 billion dollars calculating 1$=100¥.)

  (The source of both numerical data is “Life Insurance Business in Japan 2011-2012”.)

- There is no studies for mixed insurance, as far as we know. This is natural, since this insurance is specific to Japan.
- It is not impossible to rationalize why mixed insurance is traded under expected utility theory, which is the dominant analytic tool in insurance economics.

- Behavioral economics has been fully accepted as one field of economics. (Kahneman won 2002 Nobel prize)
- Many preference representations which are consistent with empirical observations are developed, and they are used to analyze economic problems. For example, loss aversion, inequality aversion, max-min EU and others.
- Insurance is not exception of this line. Researches have been grown that these representations are used in insurance context.
- Regret theory is one of this representation, which is introduced by Bell (1982) and Looms and Sugden (1982).
- Braun and Muerrman (2004) analyze insurance demand using regret theory which is a slight modified version of the original one.

2. Purpose

- This paper combines these two lines: mixed insurance and regret theory.
- This paper examines demand for mixed insurance using regret theory introduced by Braun and Muermann (2004).

- First question: has insured to choose either term or mixed insurance, when she has to choose either of them?
- Answer: For actuarially fair case,
  - Term insurance is always chosen in expected utility theory.
  - Mixed insurance may be chosen in regret theory.

→ Regret theory may justify why mixed insurance is widely traded in Japanese insurance market.

- Second question: How amount of insurance do insured purchase?

- Answer: For actuarially fair case,

In expected utility theory

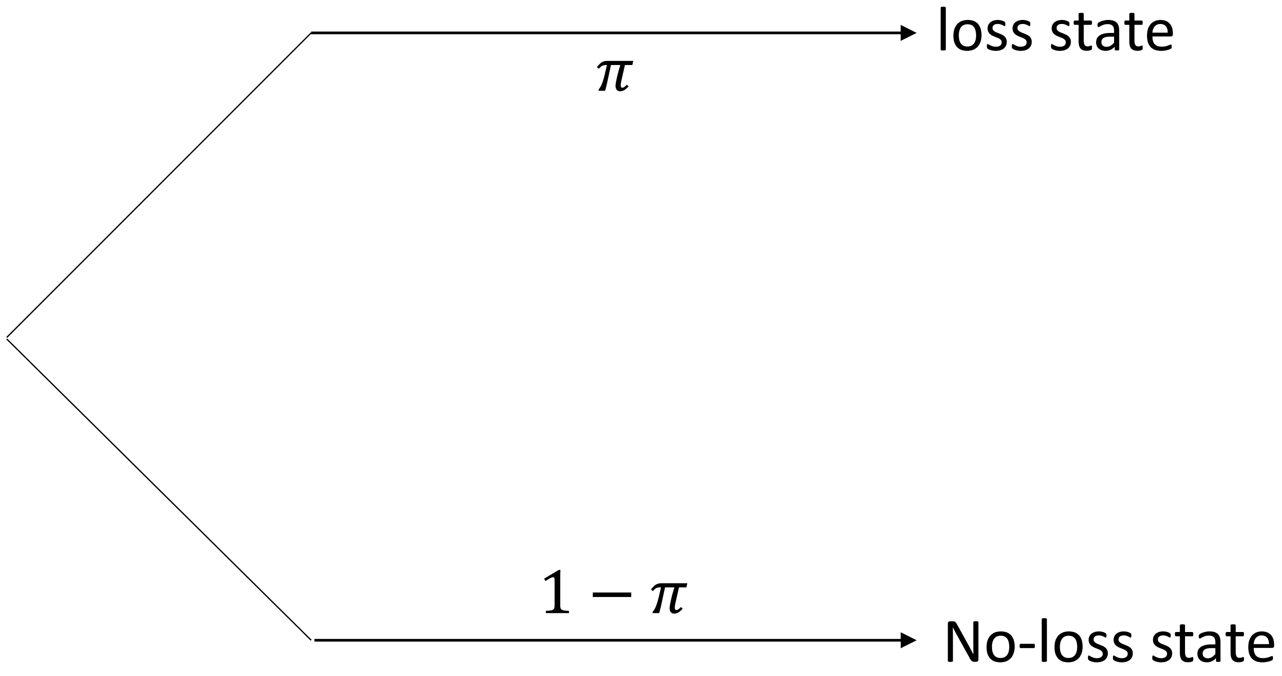
  - Full insurance is optimal in both term and mixed insurance. Full insurance is an interior solution in term insurance case, but it is a corner solution in mixed insurance case.

In regret theory

  - Partial insurance is optimal in term insurance. Both partial and full insurance can become to be optimal in mixed insurance.

3. Model

- Insured faces a situation, in which a loss state is occurred with probability π and a no-loss state is occurred with $1-\pi$.
- A loss size is $D(>0)$.



- This study considers two types on insurance, term insurance and mixed insurance.

- When insured holds one unit of term insurance, she receives coverage of D in the loss state and nothing in the no-loss state.
- When insured holds one unit of mixed insurance, she receives coverage of D in the loss state and X_{NL} ($X_{NL} \in (0, D)$) in the no-loss state.

- As to comparison, we set that both insurance premia are actuarially fair, that is,

$$P_t = \pi D \text{ and } P_m = \pi D + (1 - \pi) X_{NL}$$

We

- denote actual wealth in the loss state, x_L, and that in the no-loss state, x_{NL}.
- denote the best wealth in the loss state which can be archived, y_L, and that in the no-loss state, y_{NL}.

In this preparation, regret theoretical (RT) expected utility is represented

$$\pi[u(x_L) - g\{u(y_L) - u(x_L)\}] \\ + (1-\pi)[u(x_{NL}) - g\{u(y_{NL}) - u(x_{NL})\}]$$

- Regret can be captured by function of g, which is strictly increasing and convex. Convexity of regret function means that insured is regret averse, this assumption can be justified empirical observations *e.g.* Bleichrodt *et al*. (2010).

4. Choice

- Insured can choose to hold full insurance level of either of term insurance or mixed insurance.

In expected utility case, insured ALWAYS chooses term insurance.

Final wealth by holding mixed insurance is a mean preserving spread of that by term insurance, so that insured prefers to hold term insurance to mixed insurance by the classical result of Rothschild and Stiglitz (1971).

In other words, it is impossible to justify by expected utility theory why mixed insurance are widely distributed in Japanese insurance market, even though it is the most dominant tool in insurance economics.

In RT expected utility, insured MAY choose mixed insurance.

Intuition is as follows:

In this case, there are two effects to hold insurance, one is risk effect and the other is regret effect. Risk effect is captured by u, regret effect is captured by g.

From viewpoint of risk effect, term insurance dominates mixed insurance. However, from viewpoint of regret effect, mixed insurance may dominate term insurance. When regret effect is enough large, mixed insurance is more preferred for insured than term insurance.

The important message: regret theory may justify why mixed insurance is widely traded in Japanese insurance market.

- When insured holds term insurance, final wealth in both loss and no-loss states coincides

$$W_O = \mathrm{w} - P_t$$

- When insured holds mixed insurance, final wealth in loss state and that in no-loss state are given

$$W_L = w - P_m, W_{NL} = w + x_{NL} - P_m$$

Since $W_L < W_O < W_{NL}$, the best wealth in loss state is W_O and that is no-loss state is W_{NL}.

In this preparation, insured receives RT expected utility by holding term and mixed insurance as follows:

$$V(\text{term insurance}) = \pi[u(W_O) - g\{u(W_O) - u(W_O)\}] \\ +(1-\pi)[u(W_O) - g\{u(W_{NL}) - u(W_O)\}]$$

$$V(\text{mixed insurance}) = \pi[u(W_L) - g\{u(W_O) - u(W_L)\}] \\ +(1-\pi)[u(W_{NL}) - g\{u(W_{NL}) - u(W_{NL})\}]$$

The condition that mixed insurance is more preferable than term insurance is $V(\text{term insurance}) < V(\text{mixed insurance})$.

A necessary condition for that is given

$$\frac{g\{u(W_O) - u(W_L)\}}{g\{u(W_{NL}) - u(W_O)\}} < \frac{1-\pi}{\pi}$$

5. Demand

- First, we consider that insured follows expected utility theory.

- term insurance case

Insured's objective function

$$\max_{\alpha\in[0,1]} V(\alpha) = \pi u(w - \alpha(P_t - D) - D) + (1 - \pi)u(w - \alpha P_t)$$

It is known that $V'(1) = 0$, this means that full insurance is optimal.

- mixed insurance case

$$\max_{\alpha\in[0,1]} V(\alpha) = \pi u(w - \alpha(P_m - D) - D) + (1-\pi)u\big(w - \alpha(P_m - D)\big)$$

After some manipulations, we get $V'(1) \geq 0$. This means that full insurance is also optimal in mixed insurance case.

Full insurance is optimal in both cases, but it should be noted their difference.

Term insurance → full insurance is an interior solution.

Mixed insurance → full insurance is a corner solution. Insured should purchase over-insurance if possible, but she at most purchases full insurance by institutional constraints.

Intuition of this result :

- When insured holds mixed insurance, her wealth is a mean preserving spread compared to the case of term insurance.
- The potential demand of mixed insurance is more than the full insurance level since the insured can decrease risk in the final wealth.

- Second, insured follows RT expected utility.

- term insurance case

$$\max_{\alpha\in[0,1]} V(\alpha) = \pi\left[u(W_L) - g\left(u(W_L^{max}) - u(W_L)\right)\right] \\ +(1-\pi)\left[u(W_{NL}) - g\left(u(W_{NL}^{max}) - u(W_{NL})\right)\right]$$

$$W_L = w - \alpha(P_t - D) - D$$
$$W_L^{max} = w - (P_t - D) \leftarrow \alpha = 1$$
$$W_{NL} = w - \alpha P_t$$
$$W_{NL}^{max} = w \leftarrow \alpha = 0$$

$$V'(1) < 0 \Longrightarrow \text{partial insurance is optimal}$$

Intuition of this result:

Assuming full insurance, insured feels regret from realization of no-loss state, so that her optimal demand of term insurance is less than unity (full insurance case).

- mixed insurance case

$$max_{\alpha\in[0,1]}V(\alpha) = \pi\left[u(W_L) - g\left(u(W_L^{max}) - u(W_L)\right)\right] + (1-\pi)\left[u(W_{NL}) - g\left(u(W_{NL}^{max}) - u(W_{NL})\right)\right]$$

$$W_L = w - \alpha(P_m - D) - D$$
$$W_L^{max} = w - (P_m - D) \leftarrow \alpha = 1$$
$$W_{NL} = w - \alpha P_m$$
$$W_{NL}^{max} = w \leftarrow \alpha = 0$$

After some manipulations,

$$\text{sgn}\{V'(1)\}$$
$$= u'(W_L)[1 + g'(0)] - u'(W_{NL})\left[1 + g'\left(u(W_{NL}^{max}) - u(W_{NL})\right)\right]$$

Since

$u'(W_L) > u'(W_{NL})$

$g'(0) < g'\left(u(W_{NL}^{max}) - u(W_{NL})\right)$,

the sign of $V'(1)$ is indeterminate.

There is a possibility that either partial or full insurance is optimal.

- To examine further, we assume $u''' = 0$.

- There are two factors to determine the mixed insurance demand, one is risk effect which is captured by u, the other is regret effect which is captured by g. We call the first risk effect, the second regret effect.
- We consider that the loss probability increases.
- There is no risk effect by the assumption of $u''' = 0$.
- Regret effect increases since $u(W_{NL}^{max}) - u(W_{NL})$ increases and $g'' > 0$.
- Combining this, $V'(1)$ decreases in loss probability.
- Partial insurance in high loss probability, full insurance in low loss probability.

Intuition of this result :

- In the full insurance case, insured only feels regret from no-loss state.
- Loss probability has two opposite effects on regret
  - One is loss probability itself. This effect decreases regret when decreasing loss probability.
  - The other is regret function. This effect increases regret when decreasing loss probability.
- The first effect is constant (linearity of π) but the second effect is increasing (convexity of g). So that the second effect dominates the first effect in high loss probability, as a result, partial insurance is optimal in high loss probability.

6. Conclusion

- Regret theory may justify why mixed insurance is traded in Japanese insurance market.
- We examine term and mixed insurance demand using expected utility and regret theory.

There are some directions of future research.

- Generalization, general state model, actuarially unfair insurance premium
- Optimal insurance design by Raviv using regret theory, Can mixed insurance justify ?
- Other insurance problems specific to Japan using regret theory, or other behavioral preference representations.