Delaying Retirement Strategy with Longevity Risk and Social Welfare Maximization in China

Cui Qiang
Central University of Finance and Economics

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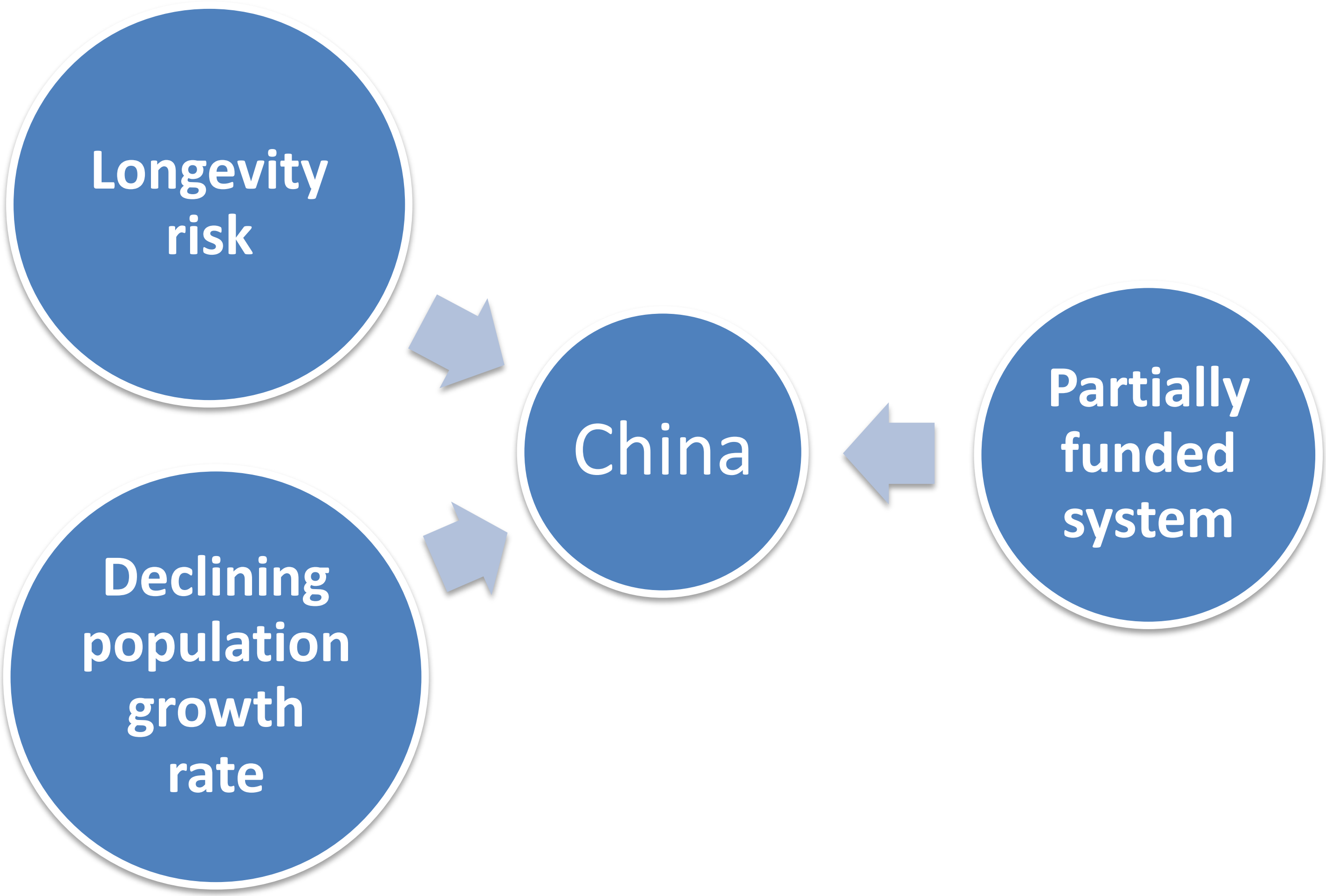
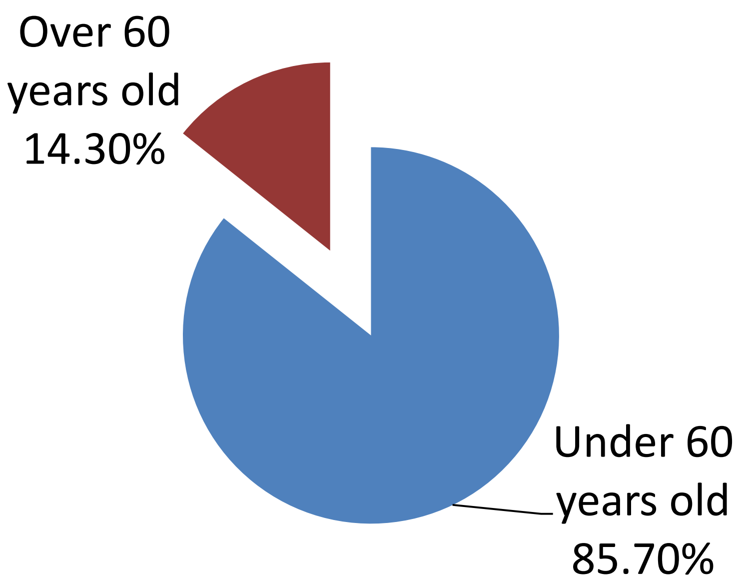
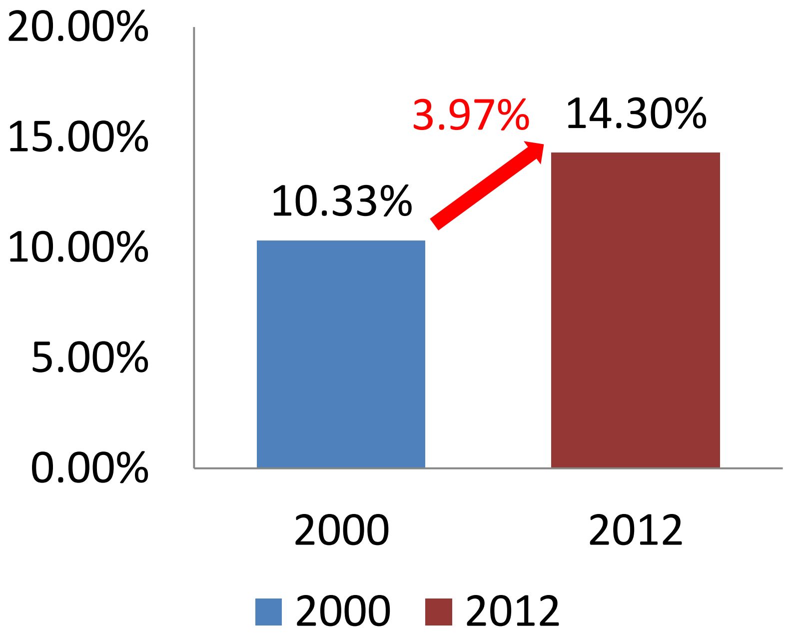
Background

Situation of China



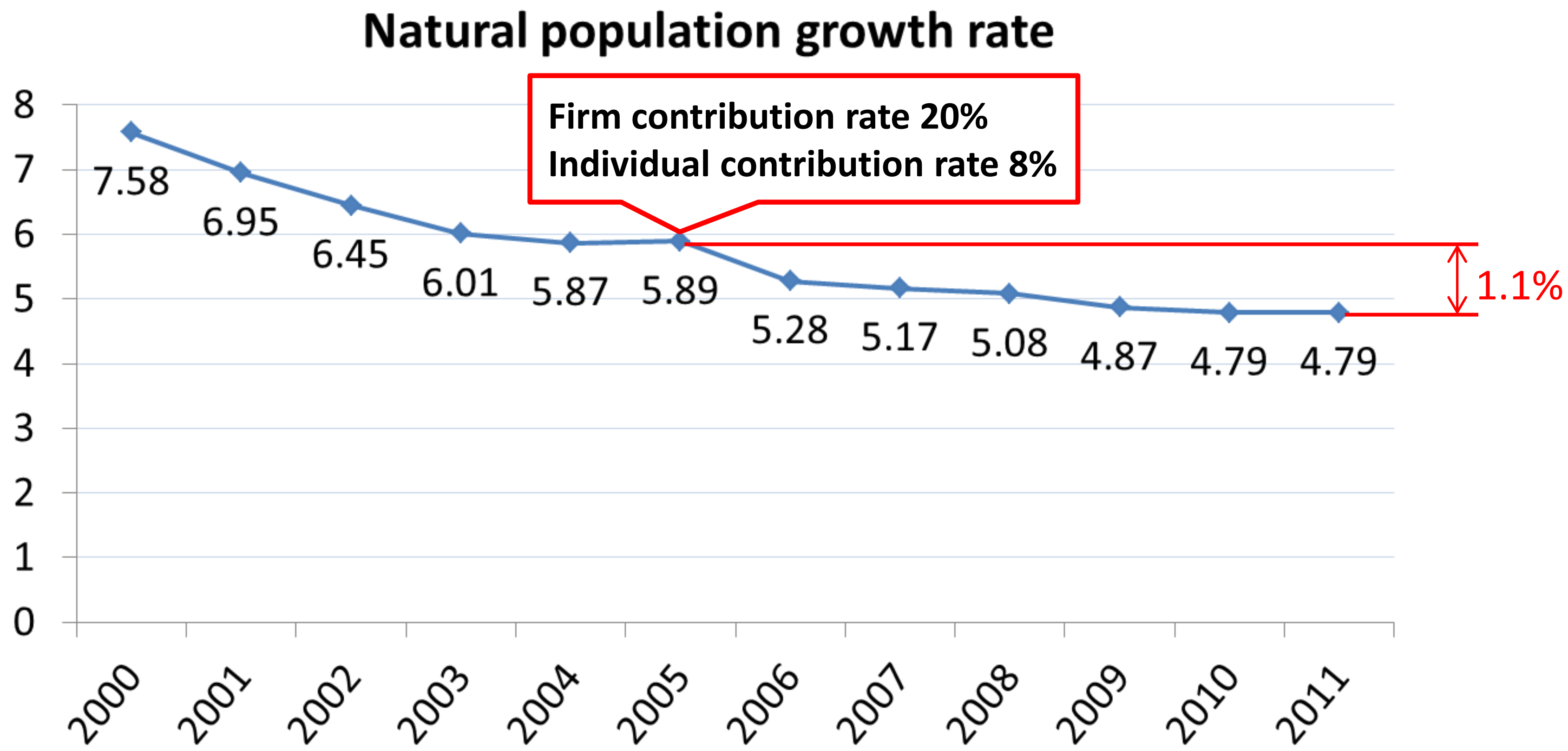
Background





The Longevity Risk

Background


Natural population growth rate
Firm contribution rate 20%
Individual contribution rate 8%
1.1%
8
7
6
5
4
3
2
1
0
7.58
6.95
6.45
6.01
5.87
5.89
5.28
5.17
5.08
4.87
4.79
4.79
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011

The Model

W_t :The wage

B_t :The bequests

$C_{1,t}$:Working-period consumption

$C_{2,t+1}$:Retirement-period consumption

(generation)
t-1

Lt-1
(population)

θW_{t+1}

I^*_{t+1}

$(1-\tau)W_t$

$(1-\theta)P_{t+1}$

t

$(1-p)B_t$

$(1+r_{t+1})S_t$

Lt

$C_{1,t}$

S_t

$C_{2,t+1}$

t+1

k_{t+1}

Lt+1

S_t : The savings

I^* : The individual account benefits (after people delay their retirement time)

P : The social pool benefits

θ : Delay retirement coefficient

Overlapping Generations model with lifetime uncertainty

The Model

Individuals

$$\max_{\{s_t\}} U_t = \ln C_{1,t} + \beta p \ln C_{2,t+1} \quad (1)$$

s. t. $$C_{1,t} = (1-\tau)W_t + (1-p)B_t - S_t \quad (2)$$

$$C_{2,t+1} = (1+r_{t+1})S_t + \theta W_{t+1} + (1-\theta)P_{t+1} + I^*_{t+1} \quad (3)$$

$$(1+n)B_{t+1} = (1+r_{t+1})S_t + I_{t+1} \quad (4)$$

(St)

$$\beta p(1+r_{t+1})C_{1,t} = C_{2,t+1} \quad (5)$$

The Model

Firm

$$Y_t = AK_t^{\alpha}(L_t + p\theta L_{t-1})^{1-\alpha}$$

- $$r_t = \alpha A G^{1-\alpha} k_t^{\alpha-1} \qquad W_t = \frac{(1-\alpha)AG^{-\alpha}k_t^{\alpha}}{1+\eta}$$

The government

- $$I_{t+1}^{*} = I_{t+1} + \theta(1+r_{t+1})\tau W_{t+1} \qquad P_{t+1} = \frac{(1+n)\eta W_{t+1} + \theta p \eta W_{t+1}}{p(1-\theta)}$$

physical capital market

- $$S_t + \tau W_t = (1+n)k_{t+1}$$

The Model

Capital market ; Firm ; Government → Equations (5)

$$\beta p(1+\alpha AG^{1-\alpha}k_{t+1}^{\alpha-1})\left[\frac{(1-\alpha)G^{-\alpha}Ak_t^{\alpha}}{1+\eta}+(1-p)(k_t+\alpha AG^{1-\alpha}k_t^{\alpha})-(1+n)k_{t+1}\right]$$

$$=(1+n)(k_{t+1}+\alpha AG^{1-\alpha}k_{t+1}^{\alpha})+[\theta+\frac{(1+n)\eta}{p}+\theta\eta+\theta\tau(1+\alpha AG^{1-\alpha}k_{t+1}^{\alpha-1})]\frac{(1-\alpha)AG^{-\alpha}k_{t+1}^{\alpha}}{1+\eta} \quad (6)$$

Dynamic Equilibrium System

Differentiating with respect to k, gives:

$$edk_{t+1}+fdk_t=0$$

The Model

Where:

$$e=\beta p\alpha(\alpha-1)AG^{1-\alpha}k^{\alpha-2}\left[\frac{1-\alpha}{1+\eta}AG^{-\alpha}k^{\alpha}+(1-p)(k+\alpha AG^{1-\alpha}k^{\alpha})-(1+n)k\right]$$

$$-\beta p(1+n)(1+\alpha AG^{1-\alpha}k^{\alpha-1})-(1-n)(1+\alpha^{2}AG^{1-\alpha}k^{\alpha-1})$$

$$-\frac{\alpha(1-\alpha)AG^{-\alpha}k^{\alpha-1}}{1+\eta}[\theta+\frac{(1+n)\eta}{p}+\theta\eta+\theta\tau+(2\alpha-1)\theta\tau AG^{1-\alpha}k^{\alpha-1}]$$

$$f=\beta p(1+\alpha AG^{1-\alpha}k^{\alpha-1})\left[\frac{(1-\alpha)\alpha AG^{-\alpha}k^{\alpha-1}}{1+\eta}+(1-p)(1+\alpha^{2}AG^{1-\alpha}k^{\alpha-1})\right]>0$$

The following conditions must be met, so that the dynamic equilibrium system has unique and stable equilibrium without oscillation:

$$0<\frac{dk_{t+1}}{dk_t}=-\frac{f}{e}<1$$

Obviously, the stability condition of the system is:

$$e+f<0$$

Pareto Optimality

Social Welfare Function:

$$E = \beta p \ln C_{2,0} + \sum_{\varepsilon=0}^{\infty} \rho^{\varepsilon} (\ln C_{1,\varepsilon} + \beta p \ln C_{2,\varepsilon+1})$$

(ρ : the social discount rate)

The resource constraint:

$$k_{\varepsilon} + AG^{-\alpha} k_{\varepsilon}^{\alpha} = (1+n) k_{\varepsilon+1} + C_{1,\varepsilon} + p \frac{C_{2,\varepsilon}}{1+n}$$

The capital stock at the beginning of next generation

Workers' maximize consumption in current generation

Pareto Optimality

Lagrangian function :

$$L = \cdots$$

$$+\rho^{t-1}(\ln C_{1,t-1} + \beta p \ln C_{2,t}) + \lambda_{t-1}\left[k_{t-1} + AG^{-\alpha}k_{t-1}^{\alpha} - (1+n)k_t - C_{1,t-1} - p\frac{C_{2,t-1}}{1+n}\right]$$

$$+\rho^{t}(\ln C_{1,t} + \beta p \ln C_{2,t+1}) + \lambda_{t}\left[k_{t} + AG^{-\alpha}k_{t}^{\alpha} - (1+n)k_{t+1} - C_{1,t} - p\frac{C_{2,t}}{1+n}\right]$$

$$+\rho^{t+1}(\ln C_{1,t+1} + \beta p \ln C_{2,t+2}) + \lambda_{t+1}\left[k_{t+1} + AG^{-\alpha}k_{t+1}^{\alpha} - (1+n)k_{t+2} - C_{1,t+1} - p\frac{C_{2,t+1}}{1+n}\right] + \cdots$$

Partially differentiating L with respect to $C_{1,t}$, $C_{2,t}$ and k_{t+1}:

$$\frac{\rho^t}{C_{1,t}} - \lambda_t = 0$$

$$\frac{\rho^{t-1}\beta p}{C_{2,t}} - \frac{\lambda_t p}{1+n} = 0$$

$$-\lambda_t(1+n) + \lambda_{t+1}(1+\alpha AG^{-\alpha}k_{t+1}^{\alpha-1}) = 0$$

$$\Rightarrow \quad k^* = G^{\frac{\alpha}{\alpha-1}}\left(\frac{1+n-\rho}{\rho\alpha A}\right)^{\frac{1}{\alpha-1}}$$

Optimal capital-labor ratio

Pareto Optimality

Dynamic Equilibrium System Equation(6)

$$k^* = G^{\frac{\alpha}{\alpha-1}} \left(\frac{1+n-\rho}{\rho\alpha A} \right)^{\frac{1}{\alpha-1}}$$

$$\beta p(1+G\cdot\frac{1+n-\rho}{\rho})[\frac{(1-\alpha)A(G\cdot\frac{1+n-\rho}{\rho\alpha A})^{\frac{\alpha}{\alpha-1}}}{1+\eta} + (1-p)\alpha A G^{\frac{2\alpha-1}{\alpha-1}}(\frac{1+n-\rho}{\rho\alpha A})^{\frac{\alpha}{\alpha-1}}$$

$$-(p+n)G^{\frac{\alpha}{\alpha-1}}(\frac{1+n-\rho}{\rho\alpha A})^{\frac{1}{\alpha-1}}] = (1+n)\left[G^{\frac{\alpha}{\alpha-1}}(\frac{1+n-\rho}{\rho\alpha A})^{\frac{1}{\alpha-1}} + \alpha A G^{\frac{2\alpha-1}{\alpha-1}}(\frac{1+n-\rho}{\rho\alpha A})^{\frac{\alpha}{\alpha-1}}\right]$$

$$+\left[\theta + \frac{(1+n)\eta}{p} + \theta\eta + \theta\tau(1+G\cdot\frac{1+n-\rho}{\rho})\right]\frac{(1-\alpha)A(G\cdot\frac{1+n-\rho}{\rho\alpha A})^{\frac{\alpha}{\alpha-1}}}{1+\eta}$$

Simulations

1.Estimation of parameter values

A period length: 30 years

Population growth rate(n): 2.481
(1974-2004)

Capital share of income (α): 0.35

Individual discount rate: 0.985 $\beta = 0.985^{30}$

Survival probability(p): 40%

Social discount rate(ρ): 0.2621

Table 1 Baseline parameter values

β	α	n	η	τ	p	ρ	A
0.6355	0.35	2.481	20%	8%	40%	0.2621	1

Simulations

2.Effect of exogenous variables

Table 2 Effect of survival probability

p	40%	42.37%	45.87%	
k	0.00420	0.00416	0.00409	↓
θ	0	0.15038	0.36448	↑
I^*	0.07834	0.09782	0.11772	↑
P	0.13880	0.15558	0.19399	↑

Simulations

2.Effect of exogenous variables

Table 3 Effect of population growth rate

n	2.481	2.425	2.369	
k	0.00409	0.00420	0.00432	↑
θ	0.36448	0.36074	0.35693	↓
I^*	0.11772	0.11664	0.11552	↓
P	0.19399	0.19216	0.18983	↓

Simulations

According to Table 2-3, we can calculate the elasticity of the endogenous variables with respect to p and n, which is shown in Table 4.

Table 4 Elasticity of endogenous variables with respect to p and n

	p	n
k	-20.37%	-119.15%
θ	1723.52%	45.46%
I^*	246.27%	40.65%
P	298.87%	41.79%

Delay retirement decisions are mainly affected by longevity risk.

Conclusions and Policy Suggestions

- **In order to ease the longevity risk, guarantee social welfare maximization in China, it is necessary to implement delay retirement policy.**

Conclusions and Policy Suggestions

- **Consider the negative effect of population growth rate on the delay retirement, the individual account and the social pool fund, we can loosen the Family Planning Policy to improve population growth rate appropriately.**

Thank you !