Longevity Risk in China: Evidence from Model Test

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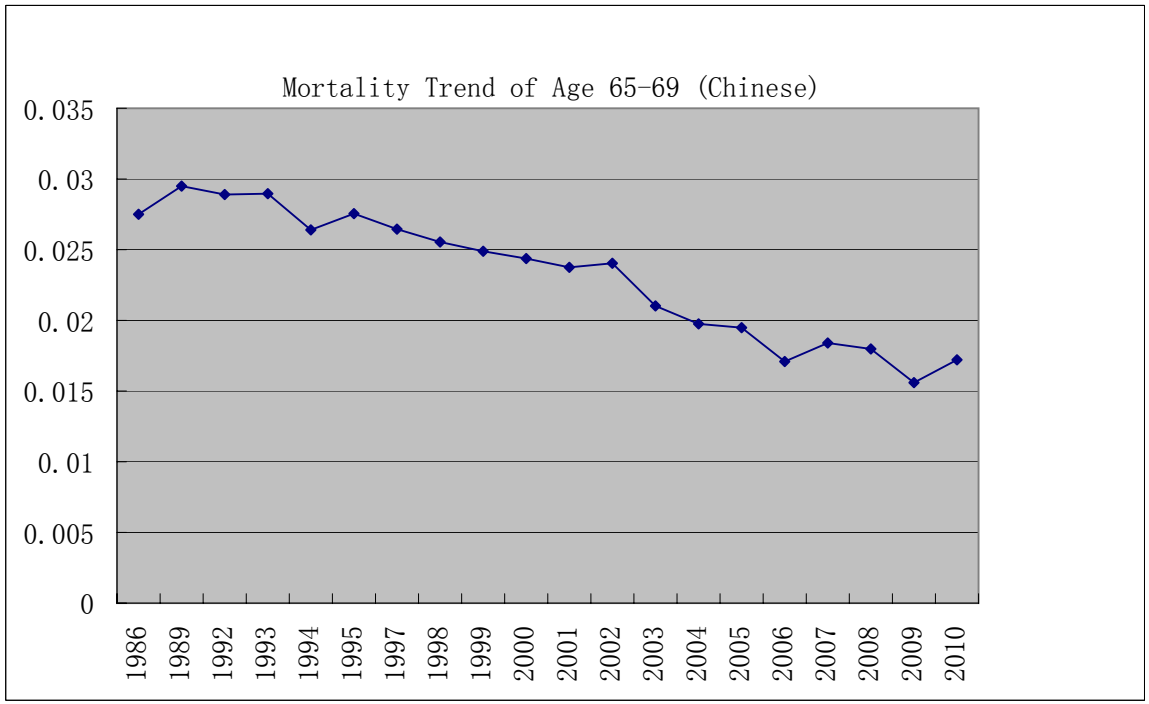
Outline of Topics

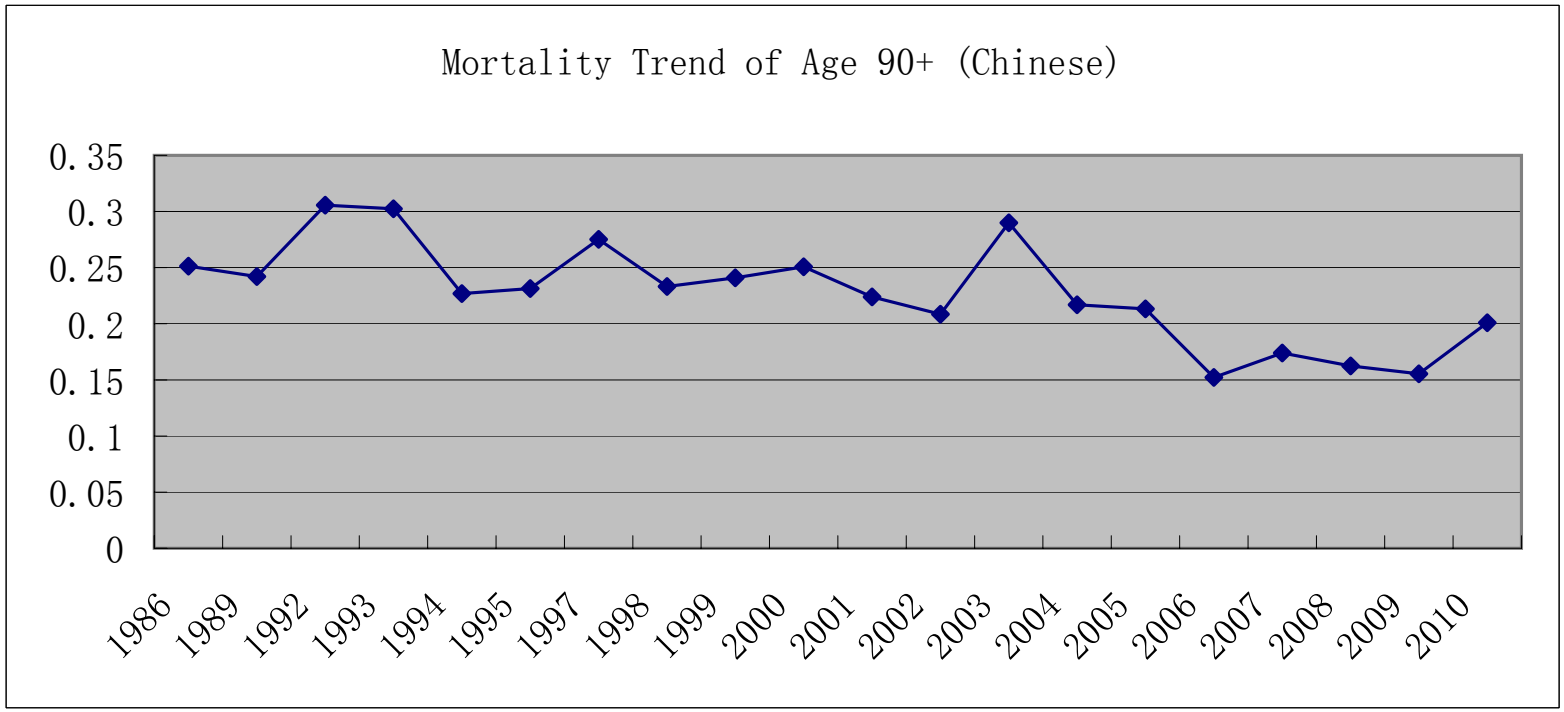
Chinese Population Data

- Missing Data: Mortality Data of Chinese population is available since 1986, missing data of year 1987, 1988, 1990, 1991, 1996
- Inconsistent structure:
  - Some provide for each age, some only available for grouped-age format (range of five years)
  - Some provide mortality data of longer age (range at 100+), some only available for 85+
  - Some are original census data, some based on sample data

Chinese Mortality Trend



Chinese Mortality Trend



Literature Review

- Chinese Case:
  - Wang & Cai (2008) concluded that the Lee-Carter model is the preferable model for Chinese data based on the analysis and comparison of different mortality models.
  - Li & Liu (2010) used Chinese data from 1992-2007 to fit the Lee-Carter Model with SVD, OLS, WLS methods and found that the WLS outperforms the other two methods.
  - Li, Lee and Tuljapurkar (2004) proposed a method to forecast mortality for populations with limited data by using Lee-Carter model.
- International Case:
  - Amato et al (2012) detected longevity common trends of multiple population

Motivation

- How to make up for the deficiency of Chinese data
- How to unify the inconsistent structure of Chinese data
- Improve the forecast ability of Lee-Carter model for Chinese Case

The Lee-Carter Model

Lee-Carter(1992) proposed a log-bilinear model for the force of mortality:

$$m_{x,t} = \exp\left(\alpha_x + \beta_x \kappa_t + \epsilon_{x,t}\right)$$

$$\log(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t} \tag{1}$$

where $m_{x,t}$ is the force of mortality of population of age x at time t

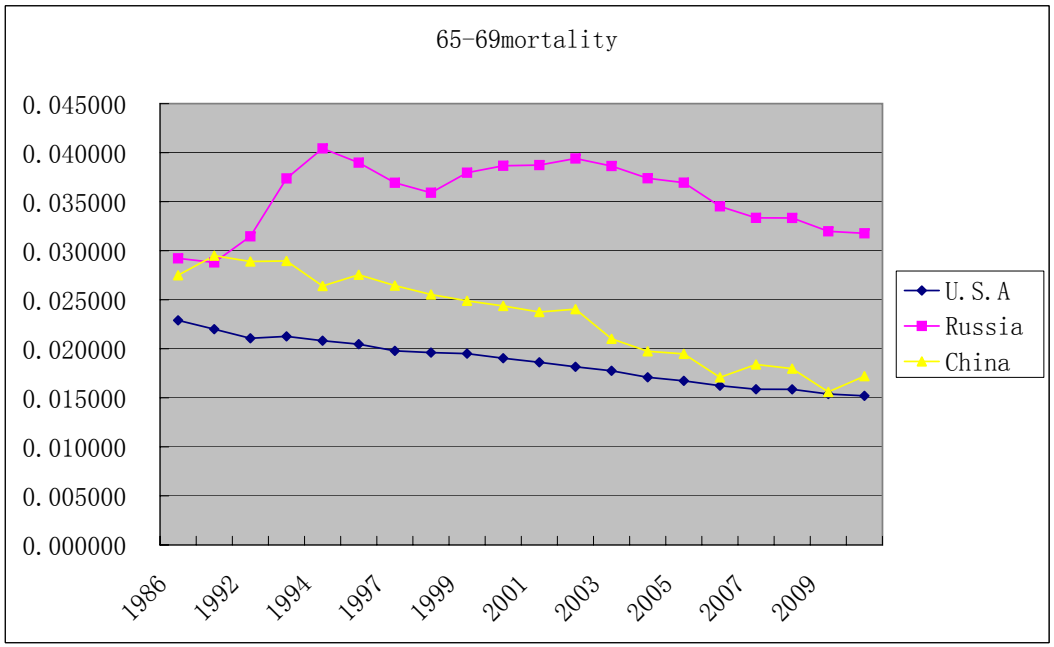
$$\sum_t \kappa_t = 0, \quad \sum_x \beta_x = 1.$$

$$\kappa_t = \kappa_{t-1} + C + \sigma\epsilon_t \tag{2}$$

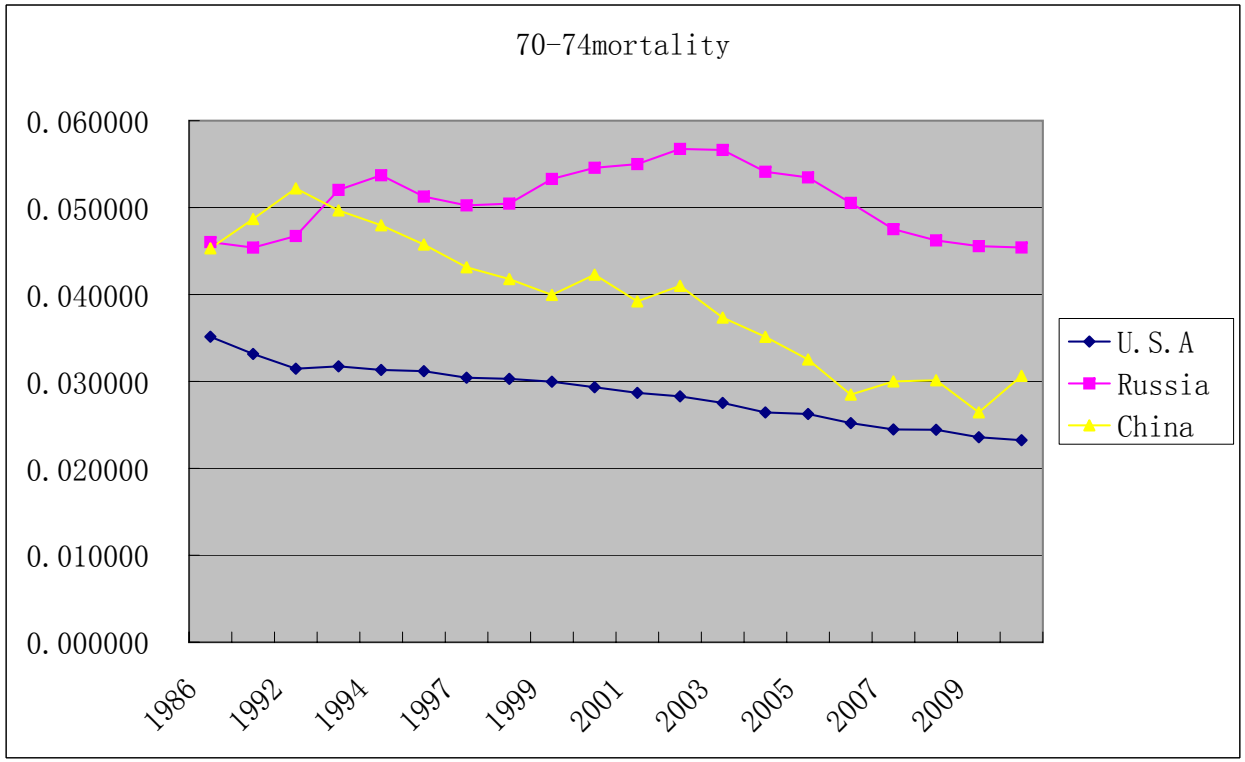
Mortality trend of Chinese case and other countries

Which country can be used to project the Chinese missing data?

- India? Russia? US?



Mortality Trend



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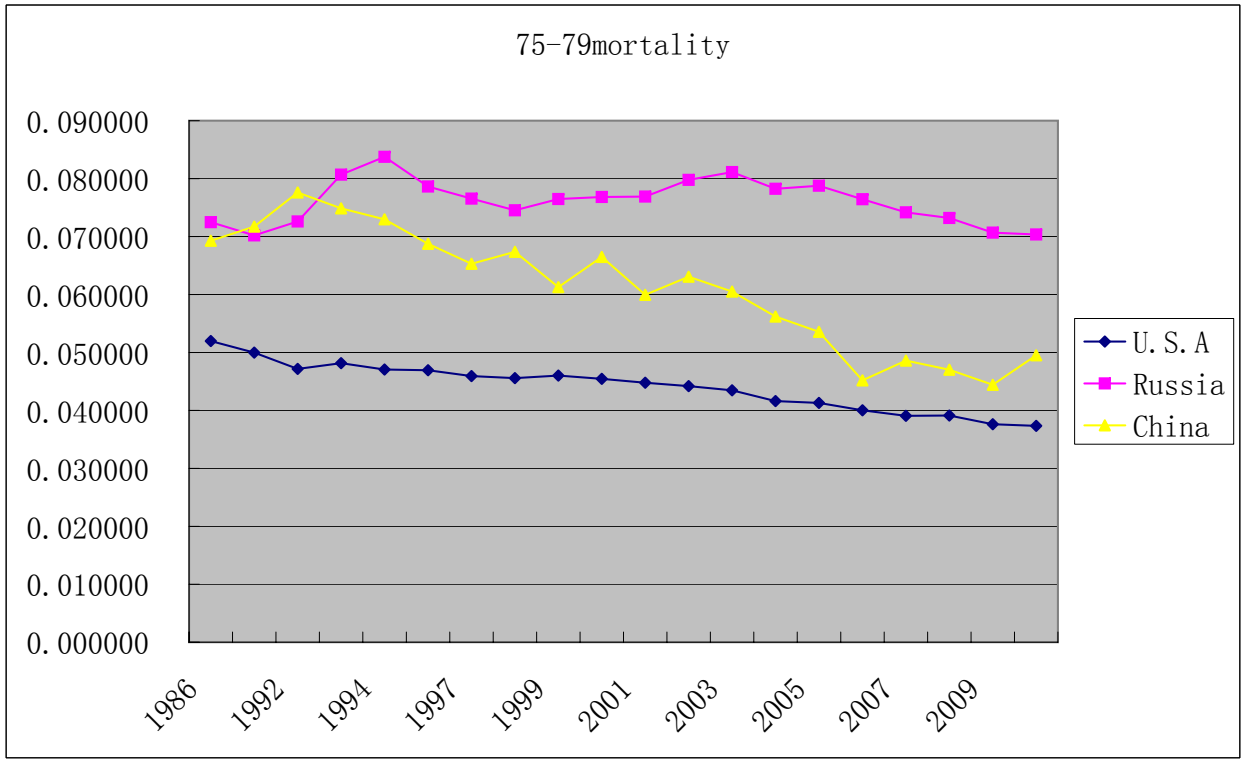

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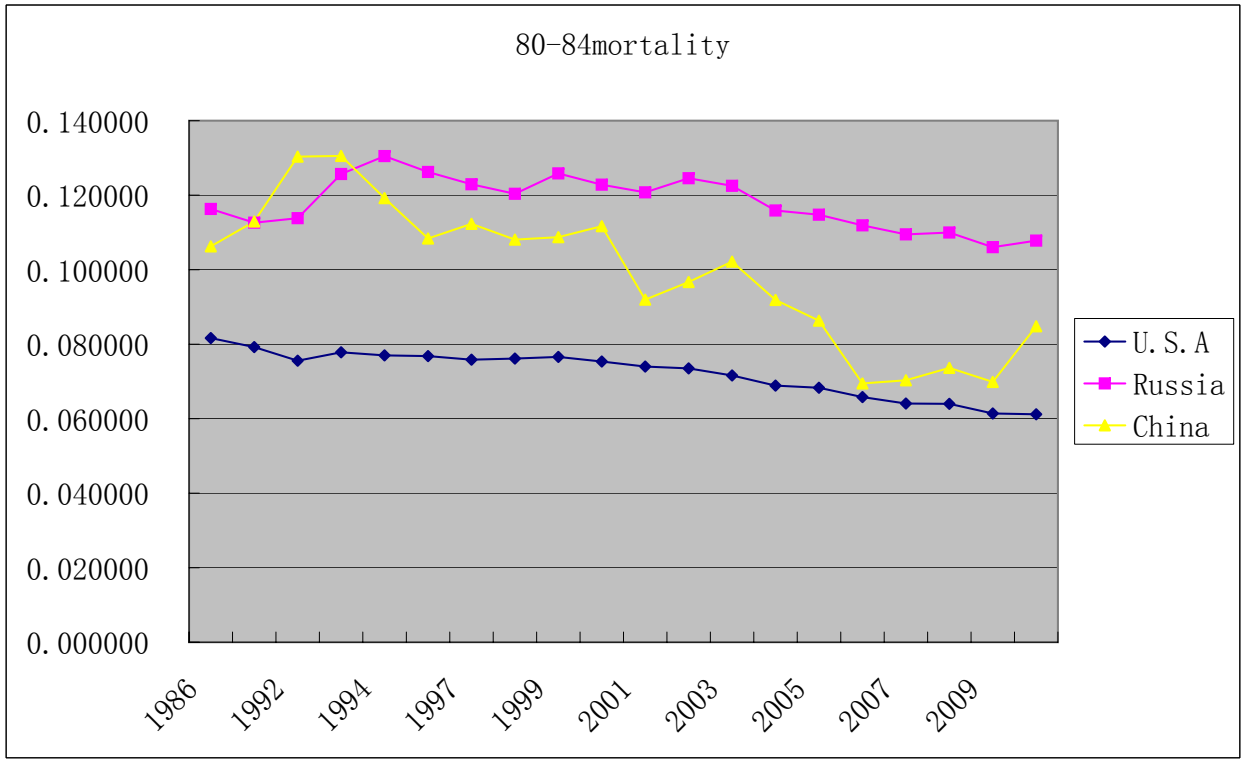

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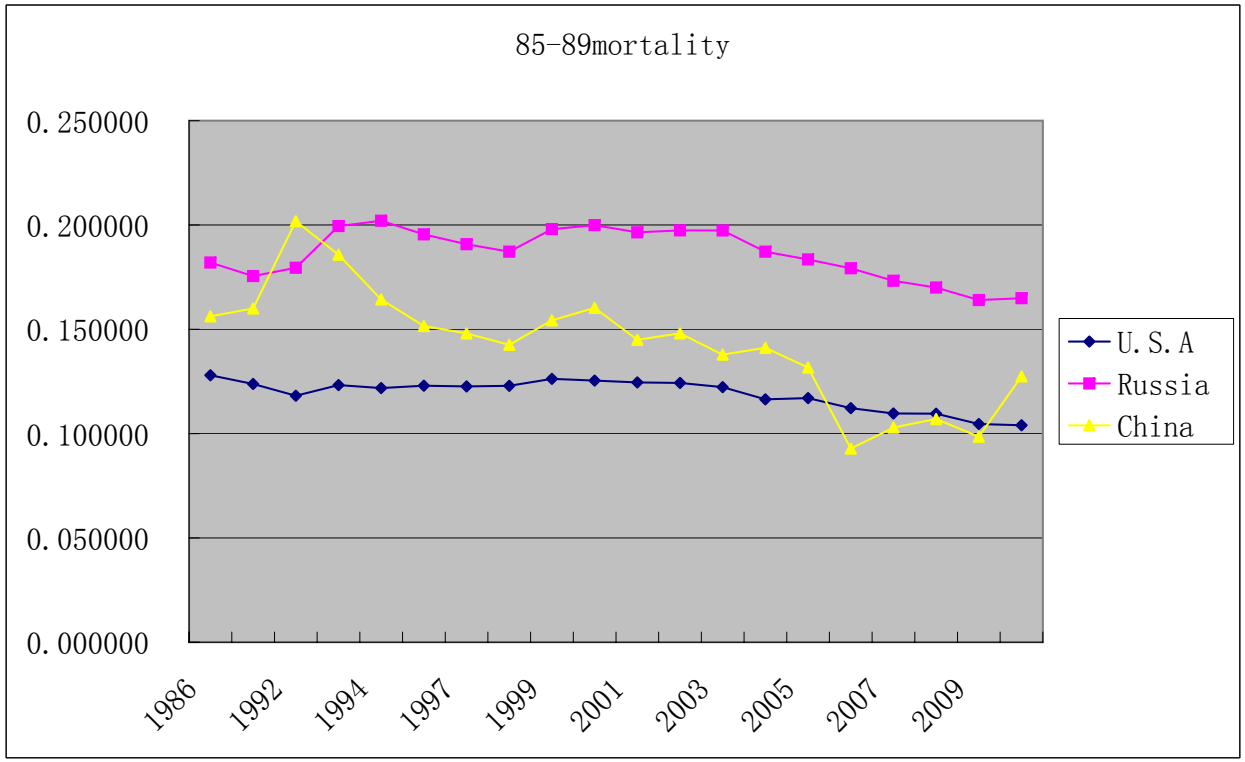

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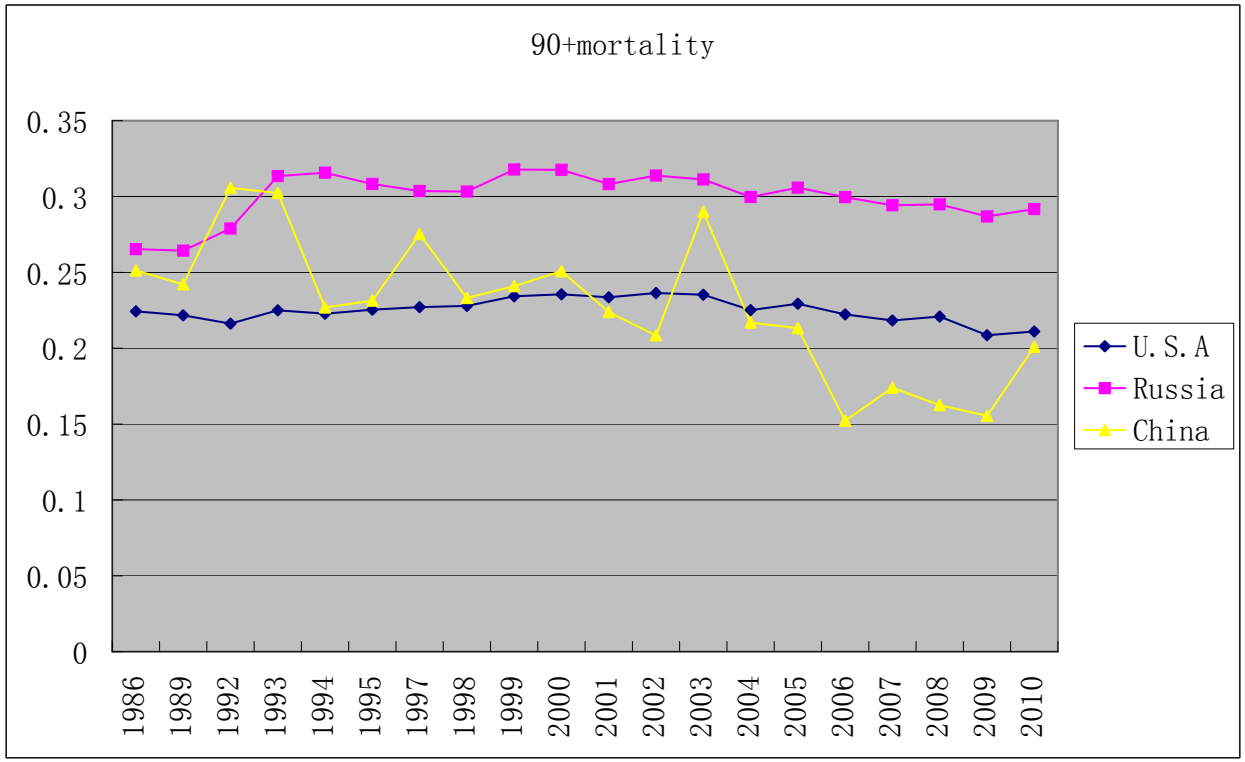


Linear Regression of mortality of two countries

- Borrowing information from one or more countries that are believed to be similar in relevant aspects. (Li, Lee & Tuljapurkar (2004))
- We propose to build a model for mortality rate of Chinese population with that of another country to find out the common trend of the two countries. If the validity of model is accepted, the missing mortality data of Chinese population will be projected from this model.
- For population of age x,

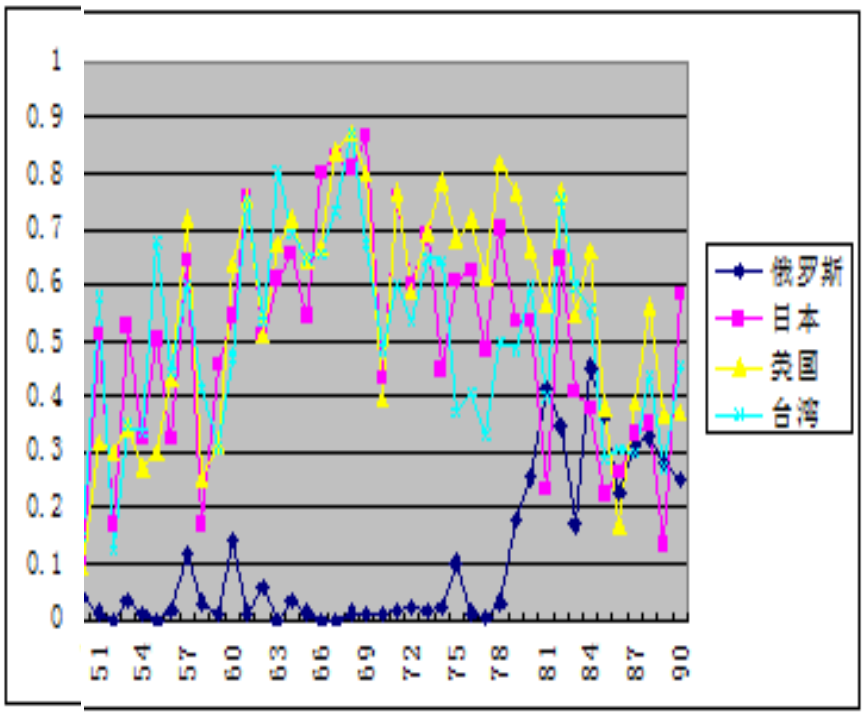
$$PRC_{x,t} = c_{x,1} + c_{x,2}X_{x,t} + \epsilon_t \tag{3}$$

where $PRC_{x,t}$ and $X_{x,t}$ are the mortality of age x at time t of Chinese population and that of another country respectively.

Goodness of Fit

- We use the mortality data of US, Japan, Taiwan to check the fit of linear correlation with that of the China.
- The weighted sum of the goodness of fit is used to choose the best fitted model.
- And the best is the US case.

Goodness of Fit



Linear Regression Result

Dependent Variable	$PRC_{65,t}$	R^2=0.887678
Intercept	-0.011407	**
$US_{65,t}$	1.854417	***

Dependent Variable	$PRC_{70,t}$	R^2=0.846972
Intercept	-0.021242	**
$US_{70,t}$	2.119014	***

Dependent Variable	$PRC_{75,t}$	R^2=0.824245
Intercept	-0.039864	**
$US_{75,t}$	2.289359	**

*** 99% significance, ** 95% significance, * 90% significance.

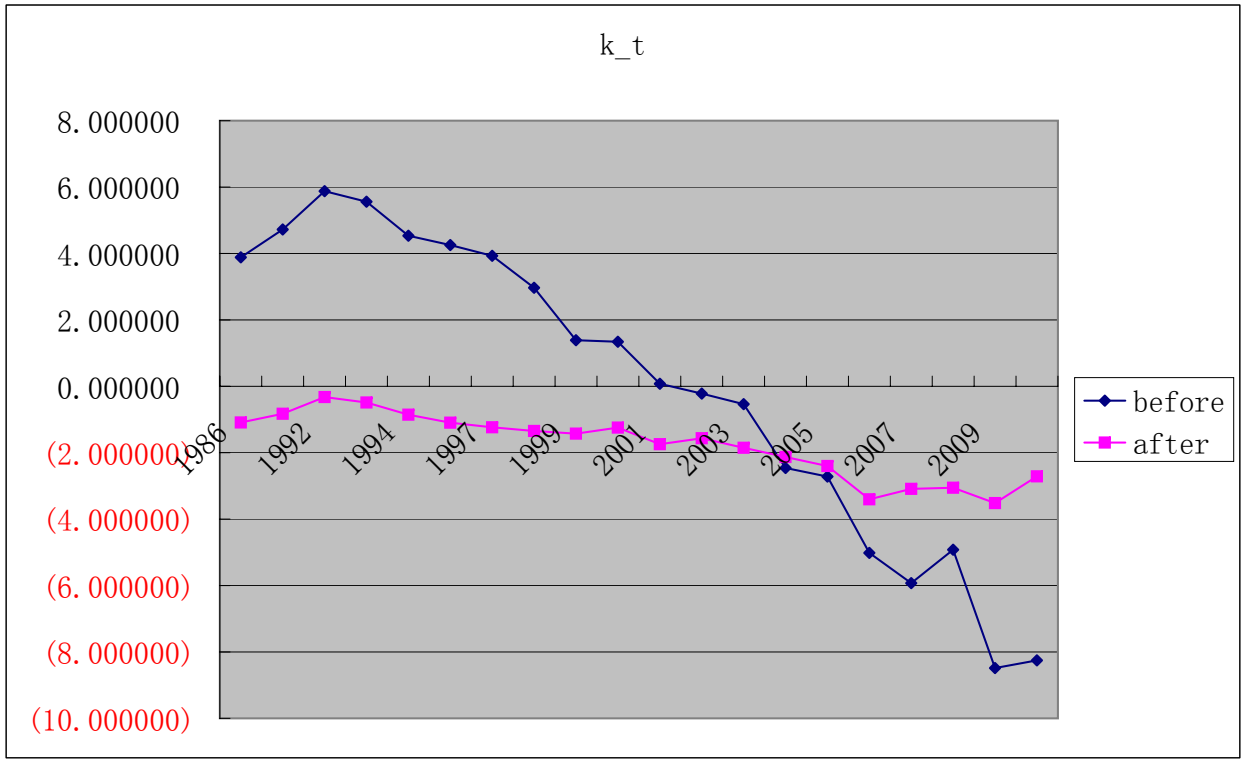
Linear Regression Result

Dependent Variable	$PRC_{80,t}$	R^2=0.729333
Intercept	-0.090740	**
$US_{80,t}$	2.630394	***

Dependent Variable	$PRC_{85,t}$	R^2=0.467645
Intercept	-0.167495	**
$US_{85,t}$	2.608392	*

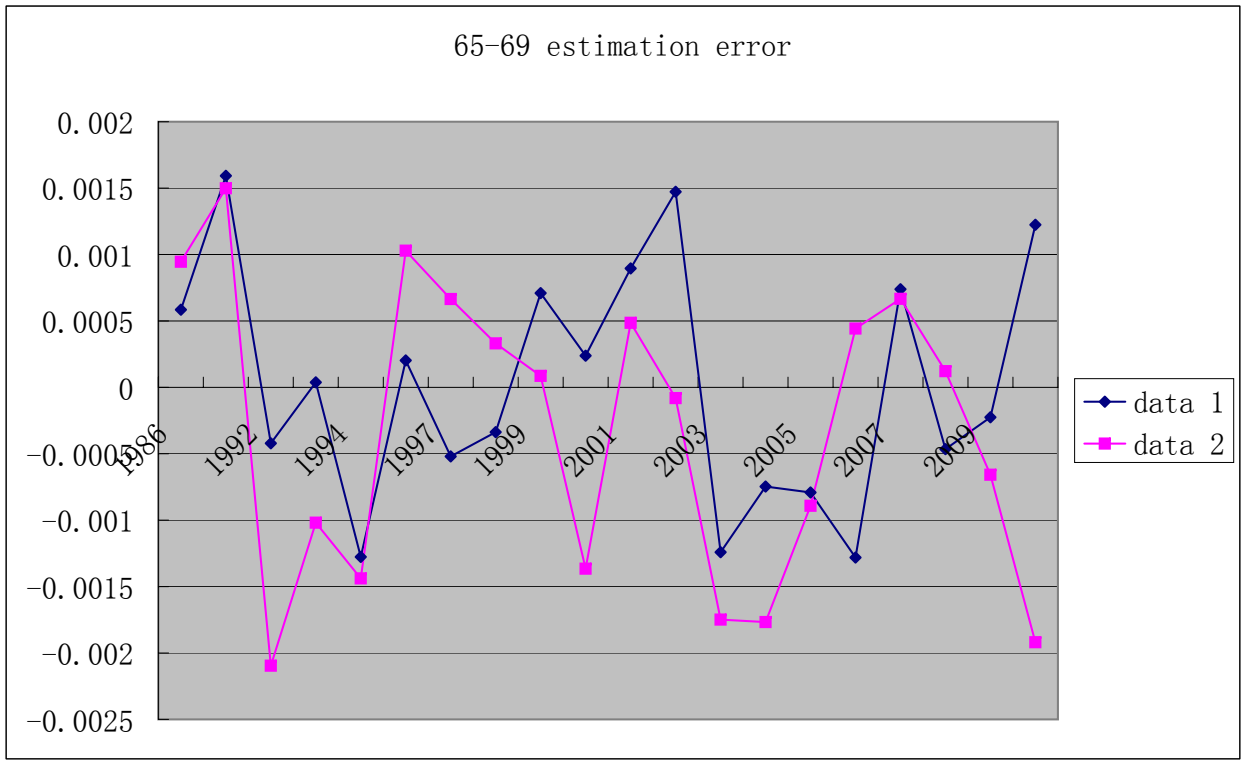
The result shows that except for the population age over 85 case, the mortality rate of the Chinese population and that of the US population has a linear correlation under the acceptable significance.

Model fitted of Lee-Carter Model



Error comparison of Lee-Carter Model with two data set

Data 1: raw data, Data 2: enhanced data with generated data from LRM



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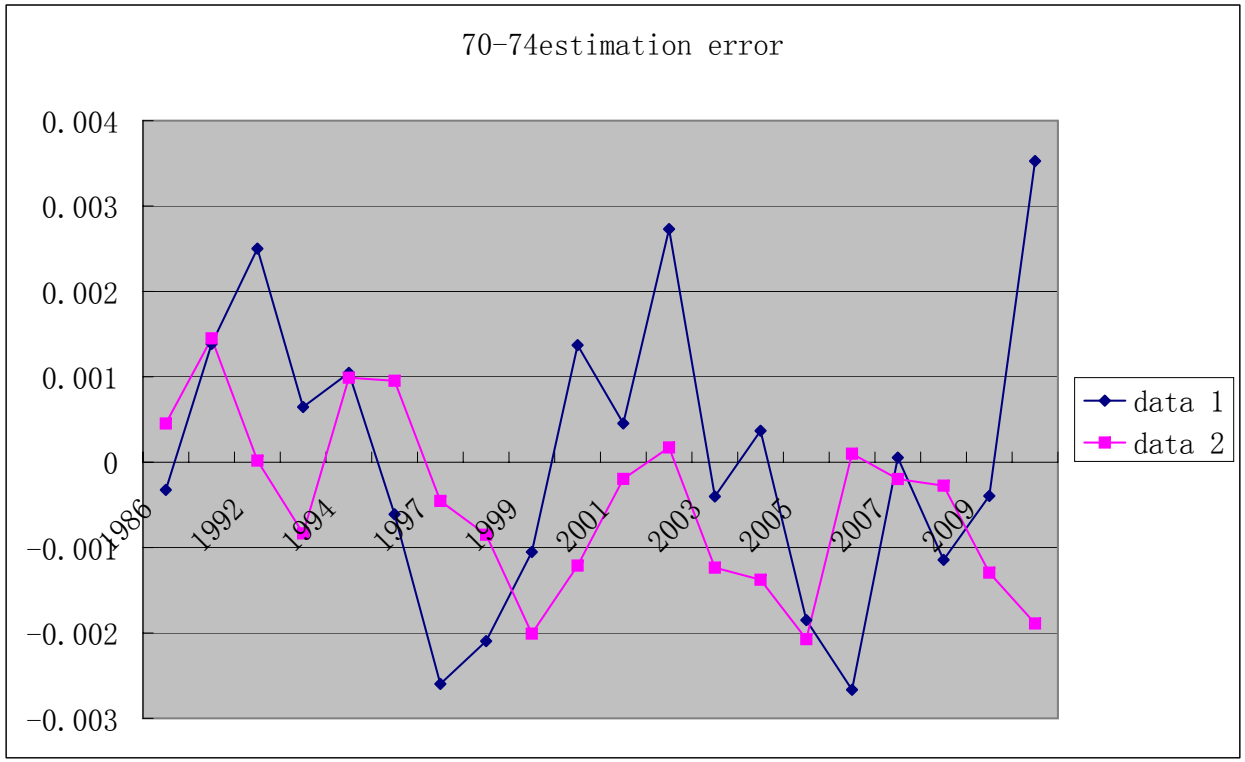

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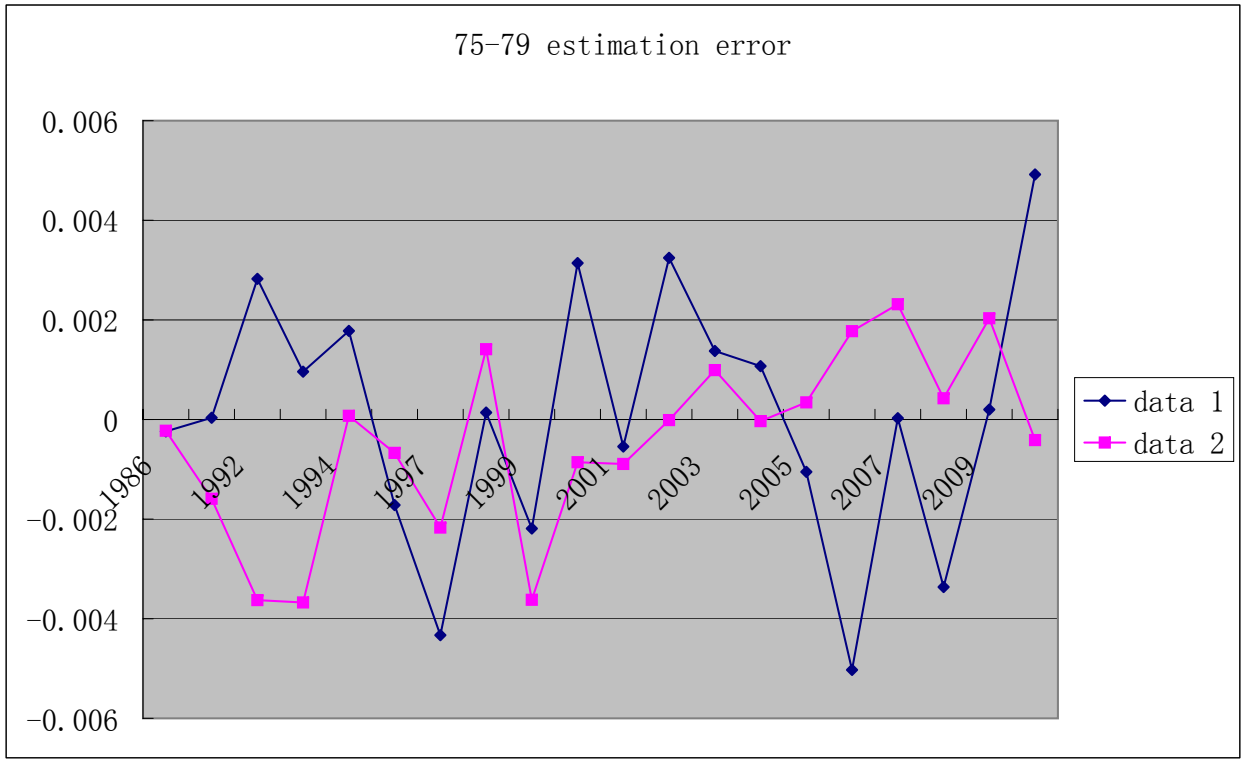

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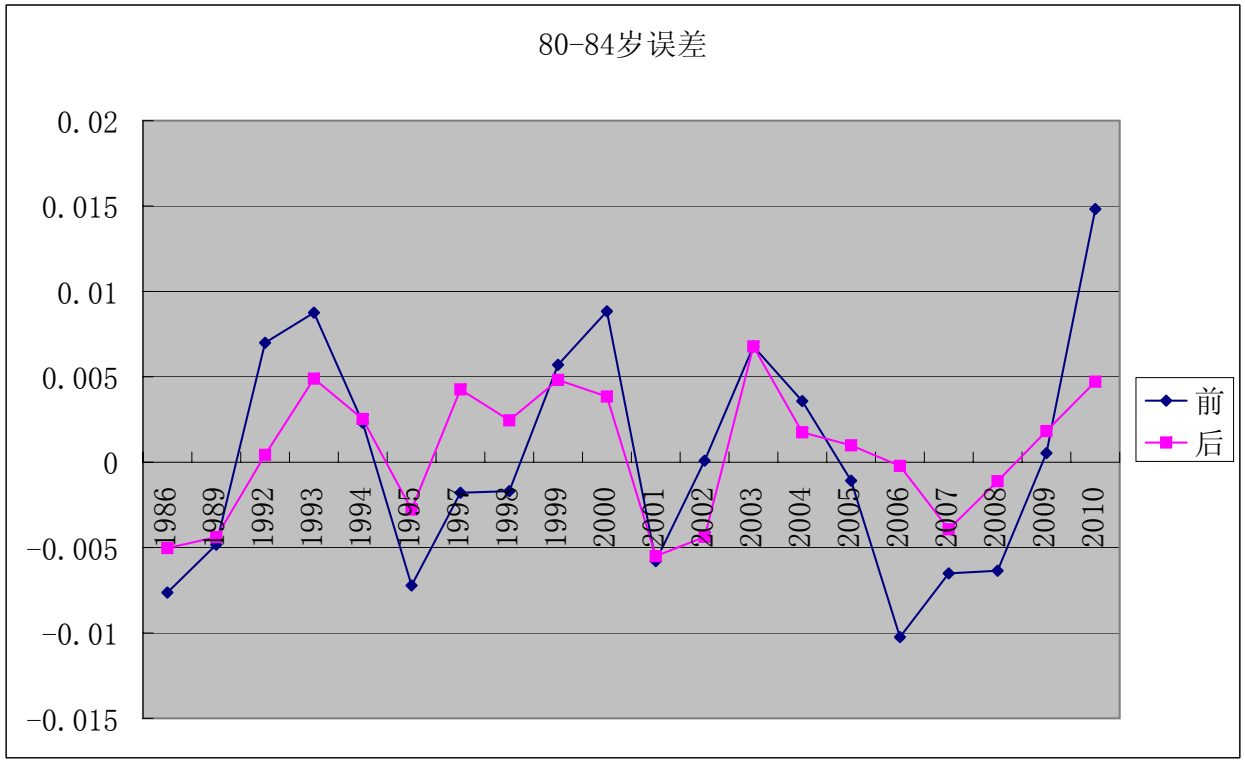

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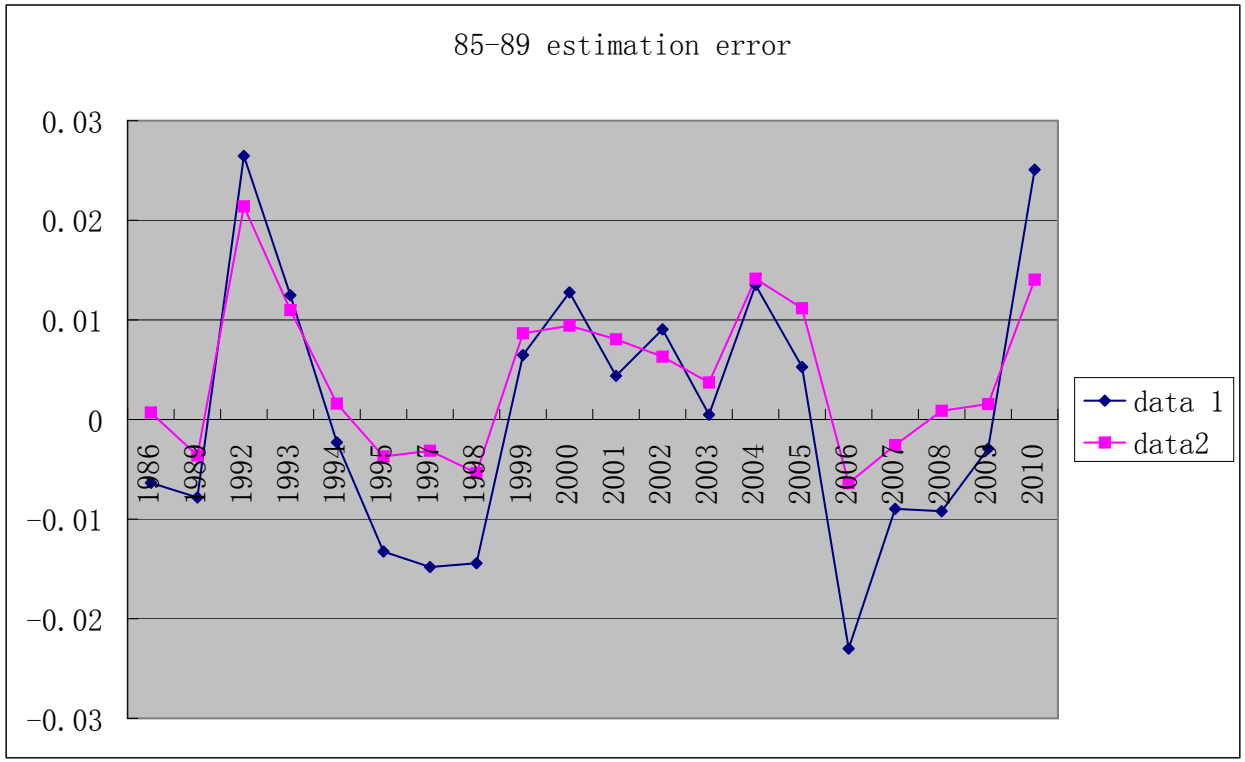

Forecasting

Using the κ_t by the enhanced data set generated by the linear model, we apply the Box-Jenkins method to find the best model for κ is the ARIMA(0,1,0) serie, which will be used to project the future mortality rate trend. The projected trend shows that mortality trends of Chinses popolation is as follows:

Age	2010	2011	2012	2013	2014	2015	2016	2017
65	14.43	13.70	13.07	12.50	11.87	11.29	10.76	10.28
70	22.48	21.07	19.97	18.59	17.66	17.48	16.63	14.72
75	42.39	40.69	39.31	38.07	36.75	35.50	34.32	32.17
80	64.47	61.89	59.48	57.03	54.61	52.24	50.30	48.12
85	92.28	88.34	84.71	81.11	77.84	74.32	71.32	68.41

Longevity Risk of China

The longevity risk is proved to exist in China, the projected mortality rate is overestimated by Lee-Carter model with the raw data. With the enhance data generated by linear regression using other country's more complete data will improved the forecast ability of Lee-Carter model.

Brouhns, N., Denuit, M. and Vermunt, J.K., (2002) A Poisson log-bilinear regression approach to the construction of projected life tables. *Insurance: Mathematics and Economics,* 31, 373- 393.

Debon, A., Martinez, F., Montes, F., (2006) Dynamic life tables: Age- period-cohort models, working paper, *Universitat de Valencia, Spain.*

Lee, R.D., and Carter, L.R., (1992) Modeling and Forecasting U.S. Mortality, *Journal of the American Statistical Association* 87, 659-71.

Li, N., and Lee, R., (2005) Coherent Mortality Forecasts for a Group of Populations: An Extension of the Lee-Carter Method, *Population Association of America* 42(3): 575-94.

Li, N., Lee, R., Tuljapukar, S., (2004), Using the Lee-Carter Method to Forecast Mortality for Populations with Limited Data, *International statistical review*, 72(1), 19-36.

 Li, H., Liu, Z., (2010) Estimation and Application of Lee-Carter Model- Based on the analysis on Chinese population data, *China Population Statistics (in Chinese)*, 46-56.

 Renshaw, A.E., Haberman, S. (2003) Lee-Carter mortality forecasting with age-specific enhancement. *Insurance: Mathematics and Economics,* 33, 255-272.

 Wang, X., Cai, Z., (2008) The recent advance on mortality models, *Statistics Research (in Chinese)*, 25(9), 80-84.

Thank you for your attention!