# **Monopoly**

## Problem 1 (APT'93, P3)

A single airline provides service from City A to City B.

- a) Explain how the airline will determine the number of passengers it will carry and the price it will charge.
- b) Suppose fixed costs for this airline increase. How will this increase in fixed costs affect the airline's price and output decisions in the short run?

#### Sample answer:

a) Being a monopolist in the market for air travels between City A and City B, the airline faces a downward sloping demand curve. As any profit-maximizing firm, the airline equates its marginal cost to its marginal revenue, so that the optimal number of passengers to carry (Q\*) is determined by the crossing of the MR and MC curves:



To decide what price to charge, the airline simply looks at the demand curve and sets the highest price the passengers are ready to pay for this optimal number of seats.

b) In the short run, the increase in fixed costs will have no effect on the airline's production and price decision whatsoever. The optimal number of passengers, the airline must carry in order to maximize its short-run profits, depends only on the position of the MC and MR curves neither of which is affected by the change in the fixed costs. As the demand curve is not affected either, the price the airline will charge for its services, remains the same and the only result of the increase in costs will be a reduction in the company's profits:



# Problem 2 (APT'94, P3)

Assume that Star Incorporated is a monopoly. Explain each of the following for this company.

- (a) Why marginal revenue and demand are not equal
- (b) How the profit-maximizing level of output and the price are determined in the short run
- (c) Why economic profits can continue to exist in the long run

### Sample answer:

- (a) In the case of a monopoly, the demand curve is downward sloping and to sell one additional unit of the good the monopolist has to cut the price on all other units. As a result, the gain from this last unit (marginal revenue) is its price minus the loss of revenues on other units of the product. That is why the monopolist's marginal revenue curve always lies below its demand curve.
- (b) To maximize short-run profits the monopolist finds the optimal output level where marginal cost is equal to marginal revenue and sets the highest price consumers are ready to pay for this quantity of the good:



(c) Since the monopoly is a sole producer of a certain good with market barriers erected to prevent other firms from entering the industry, its profits cannot attract new firms into the market and therefore do not subside even in the long run.

## Problem 3 (APT'97, P2)

An electric utility company is operating without price regulation under conditions of a natural monopoly and is currently earning economic profits.

- (a) Draw a graph and indicate each of the following for the firm.
  - (i) The equilibrium price and output
  - (ii) Economic profits
- (b) The government now wants to regulate the price. Indicate what price the government will set to achieve each of the following.
  - (i) Normal profits only (zero economic profits)
  - (ii) Efficient use of available resources
- (c) If the regulators set the price as indicated in part b(ii), will the firm continue to operate in the long run? Why or why not?

#### Sample answer:

a) The following graph shows a natural monopoly earning positive economic profits (it is characterized by a declining AC over the effective range of output):



- i) As any profit-maximizing firm, the monopoly sets marginal cost equal to marginal revenue, hence, the output will be Q\* and the corresponding price will be P\*.
- ii) Economic profits are equal to Q times P-AC, which is depicted by the area of the rectangle ABCD.
- b) i) The firm is earning normal profits only, if price equals average costs. To achieve this therefore the government must set the price at the level P<sub>1</sub>, where the demand curve crosses the average cost curve.



ii) Resources are used efficiently, if price is equal to the marginal cost of the last unit produced, hence, to ensure allocative efficiency the government will set price  $P_2$ .

c) As one can see from the above diagram,  $P_2$  is less than AC for any level of output, so if the price is set to  $P_2$  the monopolist will surely be making losses. Hence, unless it is offered a subsidy to compensate for the loss, the firm will leave the market and find a better use for its resources (in the long run, the least it can do is earn a normal return, i.e. zero profits, which is still better than staying in business).

# Problem 4 (APT'98, P3)

Some businesses often charge one group of customers a higher price than they do another group of customers for the same good or service, even though the costs of providing the good or service are the same.

- (a) Explain how the elasticities of demand for the two different groups affect the different prices they are charged.
- (b) Graphically demonstrate how the firm would determine the profit-maximizing price and output in each market.
- (c) Identify two conditions, other than different elasticities of demand, that are necessary for businesses to charge different groups of customers different prices for the same good or service.

### Sample answer:

- a) In the case of no price discrimination both groups of customers are charged the same price regardless of their price elasticity. However, since MR = P(1-1/|e|), where e is the price elasticity of demand, the group with a higher elasticity also has a higher MR. This means that the last unit sold in this segment of the market yielded more revenue than the last unit sold to the consumers with a lower elasticity of demand. Thus, if price discrimination had been possible then by simply redistributing the sales from the consumers with inelastic demand to those with elastic demand the firm could have gained extra revenue while keeping the costs constant. This explains why firms exercising price discrimination tend to sell more at a lower price to the buyers with elastic demand and charge a higher price to the customers whose demand is less elastic.
- b) It follows logically from the above considerations that the price discriminating firm will continue to draw a wedge between the prices it charges to different groups of consumers until the marginal revenue of the last unit sold is the same for the both groups. Hence, the MR curve such a firm is facing is the horizontal sum of the marginal revenue curves of the corresponding groups of buyers:



To determine the profit maximizing level we now add cost functions to the right diagram and see where the MC curve crosses the total MR curve. Then we go across to the left from this point to find out the quantities and the corresponding prices for each group of customers:



Thus, the firm will produce a total output of  $Q = Q_1 + Q_2$ , selling  $Q_1$  at price  $P_1$  to the group of consumers with inelastic demand and  $Q_2$  at price  $P_2$  to the consumers with elastic demand (note that  $P_1 > P_2$ ).

- c) The two necessary conditions that enable price discrimination are:
  - 1) The firm must be a monopolist. Otherwise consumers who are charged a higher price would be able to get it from another seller.
  - 2) The good cannot be resold (e.g. services, airline tickets, etc.) violation of this condition can lead to profiteering.

# **Problem 5 (APT'2000, P1)**



The diagram above shows the cost and revenue curves for a monopoly.

- (a) How does a monopolist determine its profit-maximizing level of output and price?
- (b) Using the information in the graph, identify each of the following for the monopolist.
  - (i) The profit-maximizing level of output and price
  - (ii) The line segment of the demand curve that is elastic
- (c) Suppose that the industry depicted in the graph became perfectly competitive without changing the demand or cost curves. Identify the equilibrium price and output that would prevail in the perfectly competitive market.
- (d) Using the information in the graph, identify the area of consumer surplus for each of the following.
  - (i) The profit-maximizing monopoly
  - (ii) The perfectly competitive industry
- (e) Define allocative efficiency.
- (f) To be allocatively efficient, what level of output should the monopolist produce?
- (g) Should the government use a per-unit tax or a per-unit subsidy to lead the monopolist to produce the allocatively efficient level of output? Explain how this tax or subsidy would achieve the allocatively efficient level of output.

### Sample answer:

- a) Just like any profit-maximizing firm a monopolist chooses the level of output where MR = MC (this condition is a straightforward result of differentiating the equation for profits with respect to output and setting the first derivative equal to zero). To determine the price the monopolist simply reads it off the demand schedule at the corresponding output level.
- b) (i) As one can see from the graph, the profit-maximizing output is Q<sub>1</sub> (where the MR curve crosses the MC curve). The price that the monopolist will charge is P<sub>5</sub>.
  (ii) In the elastic portion of the demand curve increasing output results in positive increments in the total revenue, as a reduction in price is outweighed by a greater

increase in quantity. Hence, demand is elastic where MR>0 which corresponds to the segment AD of the demand schedule.

- c) In contrast to the monopoly, under perfect competition each producer constitutes a tiny part of the market and faces a horizontal demand curve. As a result, marginal revenue of a single firm is equal to the market price and at the profit-maximizing level marginal cost is equated to the price, rather than the marginal revenue of the industry as a whole. Consequently, equilibrium will be at point C, where the MC curve intersects the demand curve. This gives us an output of  $Q_2$  and price  $P_4$ .
- d) Consumer surplus is the amount consumers would have been willing to pay for the good in excess of what they actually pay. In the case of the profit-maximizing monopoly, it is the area of the triangle ABP<sub>5</sub>. Under perfect competition consumer surplus increases to the area of the region ACP<sub>4</sub>.
- e) Allocative efficiency means utilizing scarce resources in the production of goods in such a way that it would not be possible to make someone better-off without making someone else worse-off. The benchmark of allocative efficiency is the equality P=MC which ensures that the total surplus (the sum of the consumer and producer surplus) is maximized.
- f) To be allocative efficient the monopolist must produce where P = MC, that is, at point C. The allocative efficient level of output is therefore  $Q_2$  (the same as the one provided by a perfectly competitive industry).
- g) Obviously, without any intervention the monopolist will produce below the allocative efficient level of output and a per-unit tax would only increase inefficiency pushing the monopolist farther away from the optimal point. To induce him to produce the socially desirable level of output the government should offer him a per-unit subsidy. The subsidy will shift the monopolist's cost functions down inducing him to expand the output. The size of the subsidy must be chosen in such a way that the new profitmaximizing level of production (point of intersection of the new MC and MR curves) would be  $Q_2$ . The following graph shows how it would work:



# Problem 6 (APT'95, P3)

- (a) Identify the relationship between price and marginal revenue and explain why this relationship exists for each of the following.
  - (i) A perfectly competitive firm
  - (ii) Monopoly
- (b) A firm's market power is sometimes measured by using the following formula (P-MC)

P

where P is price and MC is marginal cost at the profit-maximizing output level. Some economists claim the larger the value of the index, the greater the firm's market power.

- (i) Explain why this index is always positive for an imperfectly competitive market.
- (ii) Using the formula, calculate the market power of any perfectly competitive firm.

## Sample answer:

(a) (i) For a perfectly competitive firm price is always equal to marginal revenue. The reason for this is that a firm operating under the conditions of perfect competition constitutes a tiny part of the market, which allows it to sell as much of its output at the going market price as it wants. Hence, if the firm increases the production by one unit, the revenues will increase by the price at which the product is sold.

(ii) For the monopoly, the relationship between price and marginal revenue is a little more complicated, i.e. MR=P(1+1/|e|), where e is the price elasticity of demand. This expression is obtained by differentiating the equation for total revenues (TR=P(Q)\*Q) with respect to quantity of output (Q). It follows from this, that for the monopoly marginal revenue is always less than price. The logic behind this important result is rather simple: in contrast to firms under perfect competition, the monopoly faces a downward sloping demand curve, so that if it wants to increase its sales by one unit it has to cut the price of all previous units of the good. As a result, the gain from this additional unit will be partly or fully offset by the loss of revenues on other units. It is true also for the other forms of unperfect competition, which have the negatively sloped demand curve of the firm.

(b) (i) In order to maximize profits, firms will always set marginal revenue equal to marginal costs. But, as we have already established, for firms operating in the market structure other than perfect competition marginal revenue is less than price. Consequently, at the optimal point the difference P-MC=P-MR is greater than zero, which means that the index of market power is always positive for an imperfectly competitive firm.

(ii) In the case of perfect competition, P=MR for all levels of output. Hence, at the point of maximum profit, MC=MR=P and the index of market power takes on a zero value for any perfectly competitive firm.

## Problem 7 (APT'96, P1)

In the country of Lola, sugar had always been produced in a perfectly competitive industry until a dictator seized power and monopolized the production of sugar

(a) Draw a graph that shows the output and price the monopolist would choose to maximize profits

The people of Lola revolt, imprison the dictator, and repeal the law restricting the number of sellers of sugar.

- (b) Explain two conditions that might lead to an increase in the number of sugar sellers after the repeal of the law
- (c) Describe how an individual seller would determine the profit maximizing output level of sugar if the sugar industry were perfectly competitive
- (d) Given your answers in parts (a) and (c), is the repeal of the law likely to make sugar industry more efficient? Why? In your explanation be sure to include an explanation of economic efficiency.

### Sample answer:

a) Here we have the case of the multi-plant monopolist. In the short–run, when the number of plants remains fixed, the monopolist's MC curve will coincide with the supply curve of the previously competitive industry, which is the horizontal sum of the individual MC curves (to minimize costs of production the monopolist will equate marginal costs over all plants). In contrast to the firms in a perfectly competitive market, however, the monopolist faces a downward-sloping demand schedule and his production decision will be different from that of the competitive market, as he sets MR = MC rather than P = MC (see the graph below). In the long run, in order to cut the costs the monopolist will choose to close down some of the plants making the rest produce at their minimum efficient scale, so that his long run MC will be horizontal and equal to the minimum LAC of a typical plant. Thus, in the short run the monopolist will produce  $Q_1$  charging the price  $P_1$  and in the long run he will reduce his output to  $Q_2$  and set the price at  $P_2$ .



b) The first condition that will lead to an increase in the number of firms operating in the industry (provided that the monopolist has had enough time to shut down some of the plants) is positive economic profits. Another condition is the absence of any barriers

to entry, which is satisfied since the market was initially perfectly competitive and the law enacted by the dictator was the only restriction on the number of firms.

- c) An individual producer in a perfectly competitive industry is a price-taker, which means that the price is given to him by the market and he can sell any feasible amount of the good without affecting it. Hence, he is confronted with a horizontal demand curve at the level of the going price and MR=P. As a profit-maximizer the competitive firm will choose the output which satisfies the marginal condition, i.e. MR=MC=P. However, this condition is not sufficient as it ensures either maximum profits or minimum (or sometimes even maximum) losses, without specifying which. Thus, in order to decide whether to produce at all the firm must check whether at least variable costs are covered, that is whether P is greater than AVC. If P≥AVC then the firm will produce the output given by the marginal condition, even though it may be making losses. If P turns out to be less than the minimum AVC then the firm must shut down and leave the market or wait until the price goes up again. In the long run the profit-maximizing condition remains the same except for the fact that now the producer sets P=LMC and compares P with LAC, rather than AVC.
- d) An allocation of resources is said to be efficient if social marginal benefit from the last unit of the good produced, as depicted by the price, is equal to the social marginal cost of that unit (under this condition total surplus is maximized). Since the competitive firm always sets P=MC, in the absence of any externalities which might draw a wedge between social MC and private MC or price and marginal social benefits, the outcome provided by the perfectly competitive industry is allocative efficient. In the case of the monopoly, however, price is not equal to the marginal costs, as the firm with a monopoly power sets MC = MR < P, and although the monopolist is productive efficient (since costs are minimized) he is not allocative efficient. Producing below the socially desirable level, the monopolist causes a deadweight loss, which is the surplus that society forgoes by not utilizing its resources as effectively as it could (area ABC in the graph below):



Thus, the repeal of the law will result in a greater allocative efficiency as society will gain extra surplus in the form of the triangle ABC, which was lost after the industry was monopolized.

### Problem 8 (APT'99, P3)

- (a) Using one graph for a monopoly firm and one for a perfectly competitive firm, draw and label the demand curve and the marginal revenue curve for each of these firms.
- (b) For the perfectly competitive (a price taker) firm, explain why the relationship between demand and marginal revenue exists.
- (c) For the monopoly firm, explain why the relationship between demand and marginal revenue exists.

#### Sample answer:

a) The perfectly competitive firm is a price taker and, consequently, confronts a horizontal demand curve at the level of the market price. For the same reason, the marginal revenue curve is also horizontal and coincides with the demand curve.



The monopolist, by contrast, faces a downward sloping demand curve and the marginal revenue curve lies below it.



b) For the competitive firm the demand and marginal revenue curves coincide, because its output is too small to affect the market price. The firm is such a minor part of the industry that it can sell as much of its product at the going price as it wishes without making any difference to the situation in the market. Thus from every additional unit of good sold it receives the same marginal revenue which is simply the price: MR = P.

0

c) The demand curve that the monopolist is facing is not horizontal, as in the case of perfect competition, but has a negative slope, indicating the firm's ability to alter the price. Hence, to sell one more unit of good the firm will have to cut the price, which, in the absence of price discrimination, means that the price must be lowered for all units of output. As a result, the gain from the last unit of good (its price) will be partially or fully compensated by the loss of revenue on the previous units, which explains why the marginal revenue curve always lies below the demand curve.

# **Problem 9**

Monopoly operates under the following cost and demand conditions:  $TC = \frac{1}{6}Q^2$  and Qd = 150-3P. Calculate:

a) profit-maximizing output and price;

b) economic profit of the firm;

c) consumer surplus;

d) dead weight loss of the monopoly;

e) Lerner index.

Use a correctly labeled graph to illustrate the calculations.

#### Solution:

a)  $P=50-\frac{1}{3}Q$ ;  $MR=50-\frac{2}{3}Q=MC=\frac{1}{3}Q$ ;  $Q^{*}=50$ ;  $P^{*}=33^{1}/3$ ;  $MC=16^{2}/3$ . b) PR=1250. c)  $CS=0,5(50-33^{1}/3)50=416^{2}/3$ . d)  $DWL=0,5(33^{1}/3-16^{2}/3)(75-50)=207,5$ . e)  $L = \frac{(33\frac{1}{3}-\frac{50}{3})}{33\frac{1}{3}} = 0,5$ .



### Problem 10

A profit maximizing monopoly supplies product to two segments of market demand with the following demand curves:  $P_1=144-0,75Q_1$  and  $P_2=43-1,25Q_2$ . The total cost curve is TC=4Q+0,25Q<sup>2</sup>, where Q is the aggregate output of the monopoly supplied to both segments of the market.

What price will be set and what output will be produced by the monopoly at each segment of the market.

Use a correctly labeled graph to illustrate the calculations.

#### Solution:

$$\begin{split} &\mathsf{MR}_1 = 144 - 1,5\mathsf{Q}_1; \,\mathsf{MR}_2 = 43 - 2,5\mathsf{Q}_2; \,\mathsf{MC} = 4 + 0,5(\mathsf{Q}_1 + \mathsf{Q}_2). \\ & \left\{ \begin{array}{l} 144 - 1,5\mathcal{Q}_1 = 4 + 0,5(\mathcal{Q}_1 + \mathcal{Q}_2) \\ 43 - 2,5\mathcal{Q}_2 = 4 + 0,5(\mathcal{Q}_1 + \mathcal{Q}_2) \\ 43 - 2,5\mathcal{Q}_2 = 4 + 0,5(\mathcal{Q}_1 + \mathcal{Q}_2) \\ \end{array} \right\} \\ & \left\{ \begin{array}{l} 3\mathcal{Q}_2 + 0,5\mathcal{Q}_1 = 39 \\ \mathcal{Q}_1 = 70 - 0,25\mathcal{Q}_2 \\ \end{array} \right\} \\ & \left\{ \begin{array}{l} 3\mathcal{Q}_2 + 35 - 0,125\mathsf{Q}_2 = 39; \, 2,875\mathsf{Q}_2 = 4; \, \mathsf{Q}_2 = 1,39; \, \mathsf{Q}_1 = 69,6525; \\ \mathsf{P}_1 = 144 - 0,75 \cdot 69,68 = 91,74; \, \mathsf{P}_2 = 43 - 1,25 \cdot 1,28 = 41,2625. \\ \end{array} \right. \end{split}$$

