Unit 3: Elasticity

In accordance with the APT programme the objective of the lecture is to help You to comprehend and apply the concepts of elasticity, including calculating:

- price elasticity of demand;
- cross-price elasticity of demand;
- income elasticity of demand;
- price elasticity of supply.

Required reading

Mankiw, N.G. Principles of Microeconomics. 6th edition. South-Western. 2009.

Chapter 5. Elasticity and Its Application.

Questions to be revised

- ✓Demand schedule and the law of demand;
- ✓Factors of demand, complements and substitutes;
- ✓Supply schedule and the law of supply;
- ✓Market equilibrium: welfare aspects of government controls.

Price Elasticity of Demand

Percent change in quantity demanded that occurs in response to one percent change in price:

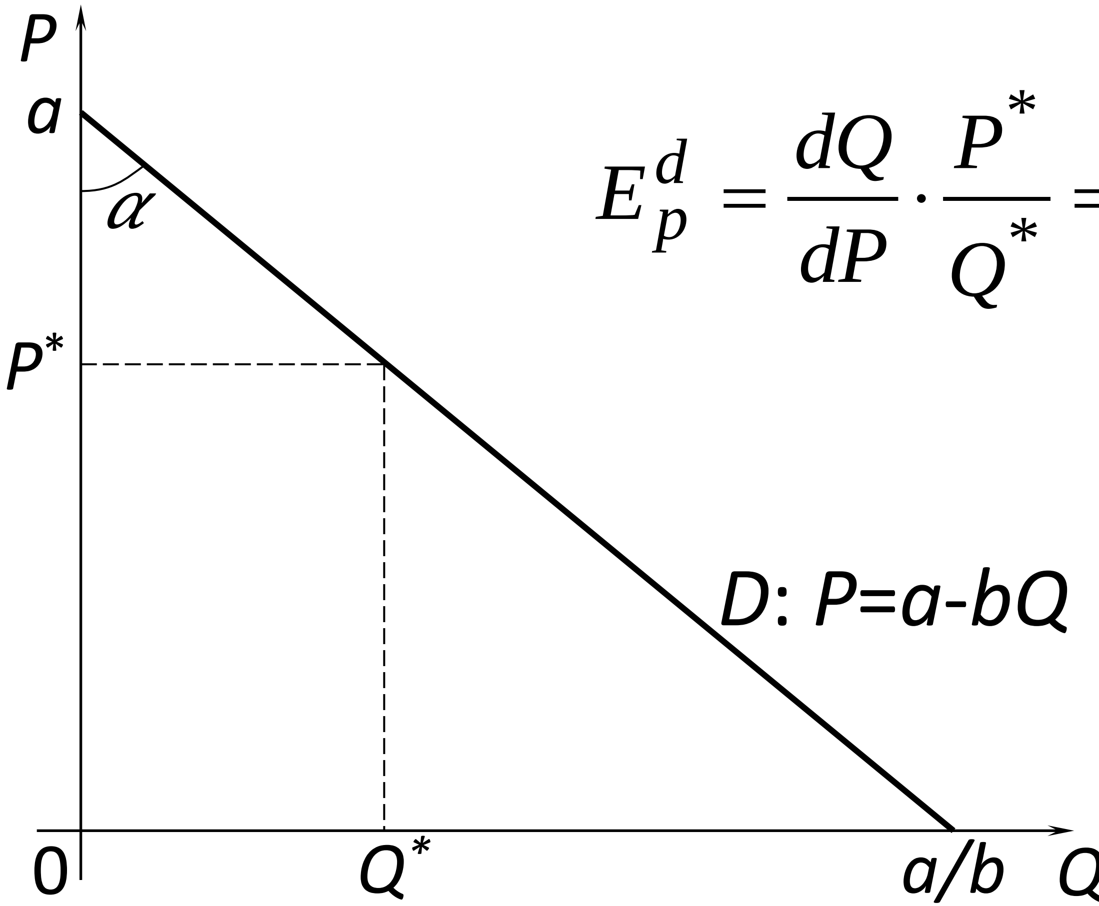
$$E_p^d = \frac{(Q_2 - Q_1)}{Q_1} \Bigg/ \frac{(P_2 - P_1)}{P_1} = \frac{\Delta Q}{Q} \Bigg/ \frac{\Delta P}{P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

Price Elasticity of Demand

Price elasticity may be defined with respect to infinitesimal changes in price:

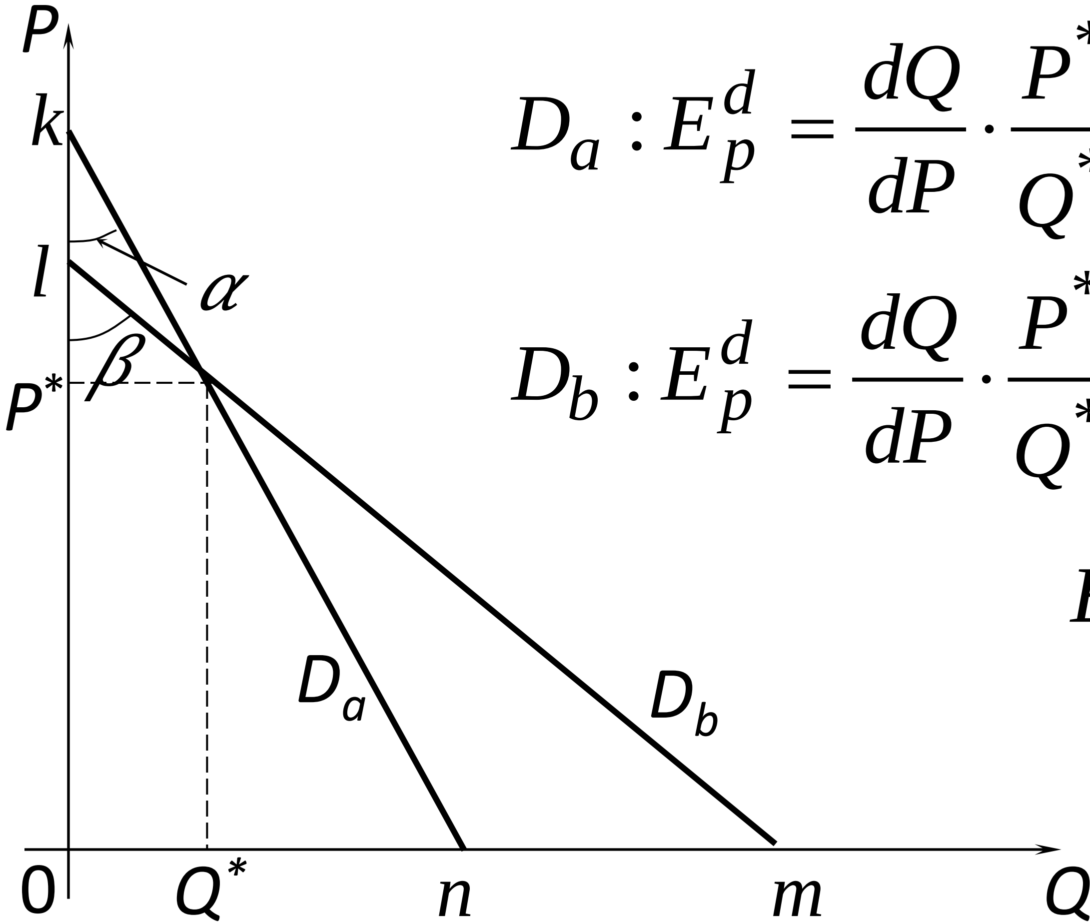
$$E_p^d = \lim_{\Delta P \to 0}\left(\frac{\Delta Q}{\Delta P}\cdot\frac{P}{Q}\right) = \frac{dQ}{dP}\cdot\frac{P}{Q}$$

Price Elasticity of Demand



$$E_p^d = \frac{dQ}{dP} \cdot \frac{P^*}{Q^*} = -tg\alpha \cdot \frac{P^*}{Q^*}$$

Elasticity and Slope of Demand Curve



$$D_a : E_p^d = \frac{dQ}{dP}\cdot\frac{P^*}{Q^*} = -tg\alpha\cdot\frac{P^*}{Q^*} = -\frac{n}{k}\frac{P^*}{Q^*}$$

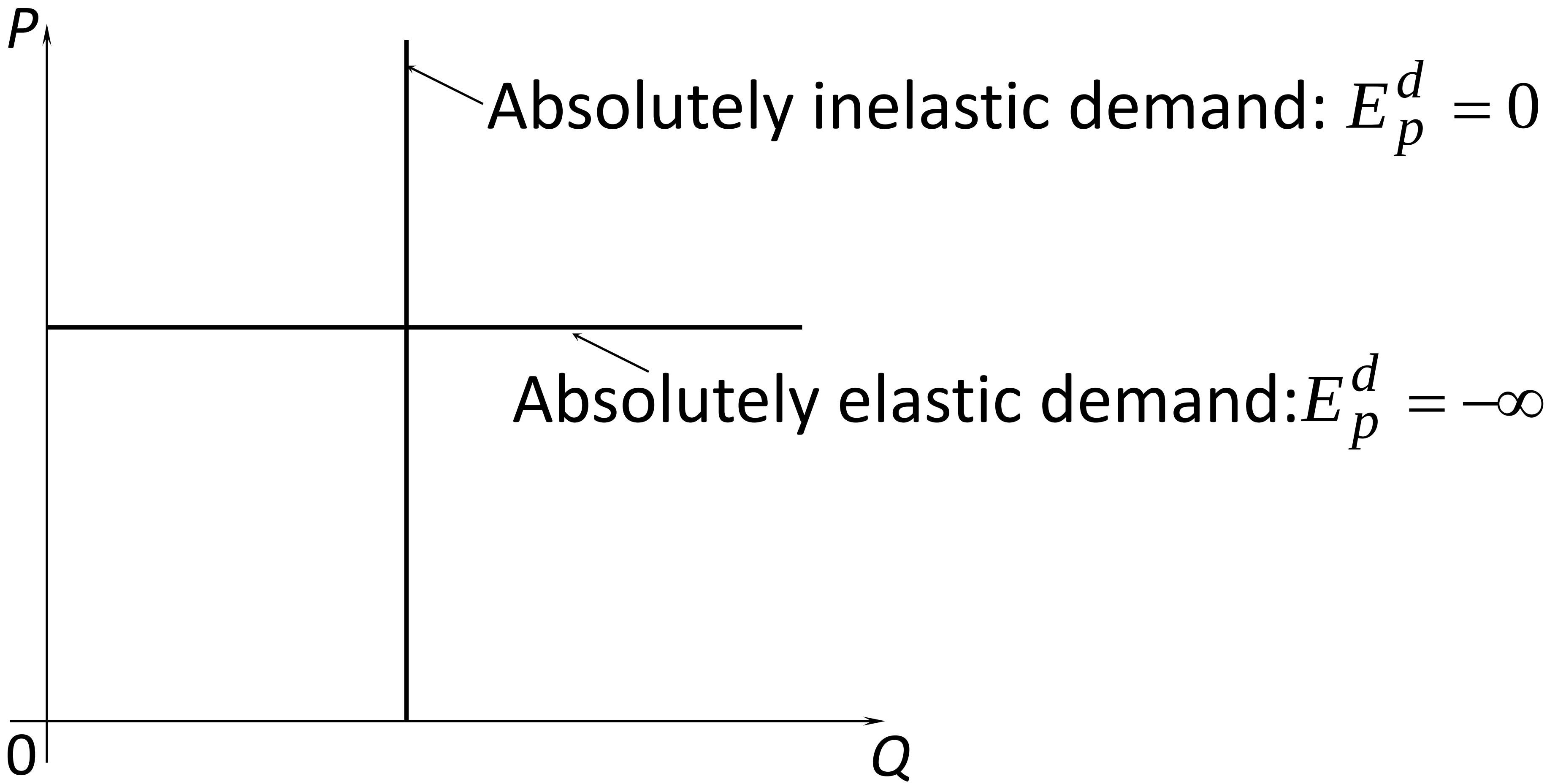
$$D_b : E_p^d = \frac{dQ}{dP}\cdot\frac{P^*}{Q^*} = -tg\alpha\cdot\frac{P^*}{Q^*} = -\frac{m}{l}\frac{P^*}{Q^*}$$

$$E_p^{D_a} < E_p^{D_b}$$

D_b is more elastic than D_a at the point (P^*, Q^*).

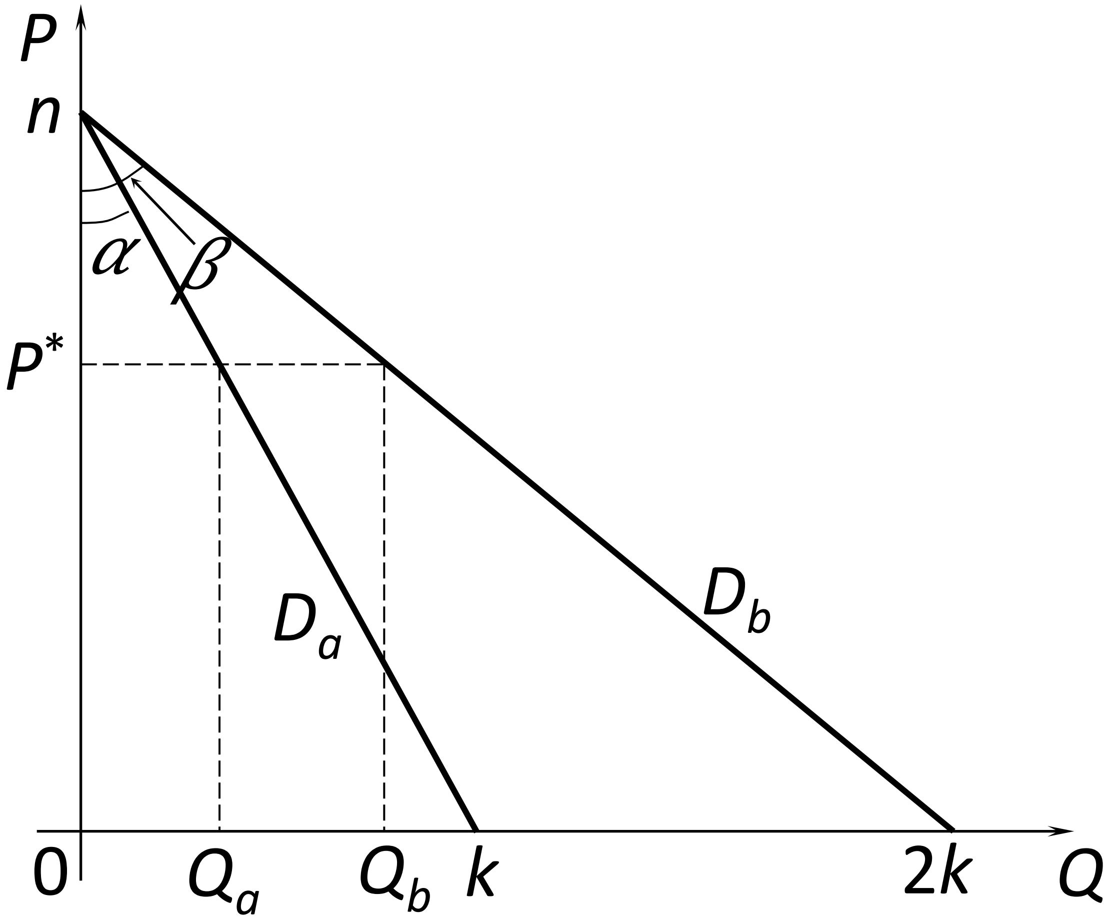
At one and the same point (P^*, Q^*) a flatter demand curve is more elastic.

Elasticity and Slope of Demand Curve



Elasticity and Slope of Demand Curve

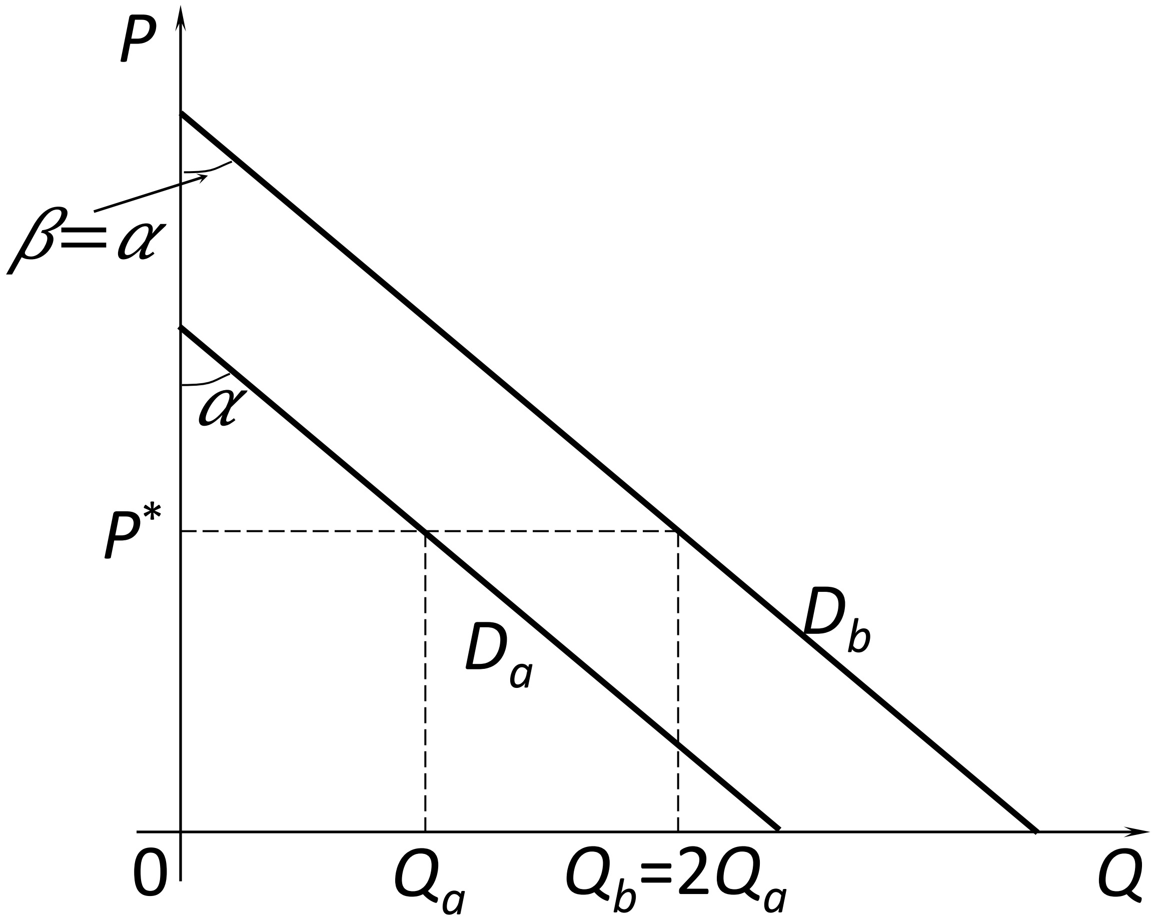
Equal elasticities and different slopes



$$E_p^{D_b} = -tg\beta \cdot \frac{P^*}{Q_b} = -\frac{2k}{n} \cdot \frac{P^*}{Q_b} = -2tg\alpha \cdot \frac{P^*}{2Q_a} = -tg\alpha \cdot \frac{P^*}{Q_a} = E_p^{D_a}$$

Elasticity and Slope of Demand Curve

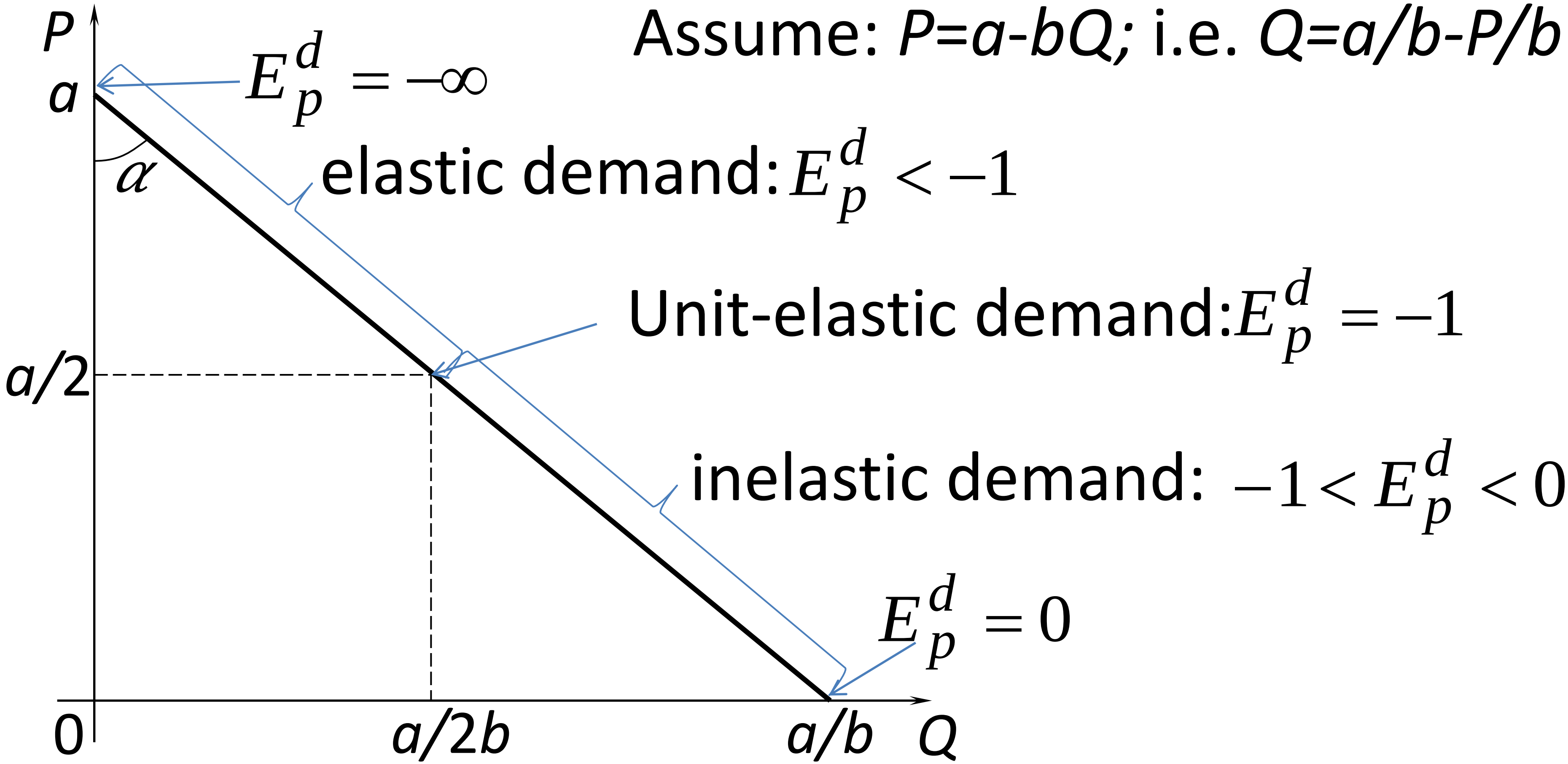
Different elasticities and equal slopes



$$E_p^{D_b} = -tg\beta \cdot \frac{P^*}{Q_b} = -tg\alpha \cdot \frac{P^*}{2Q_a} = \frac{E_p^{D_a}}{2}$$

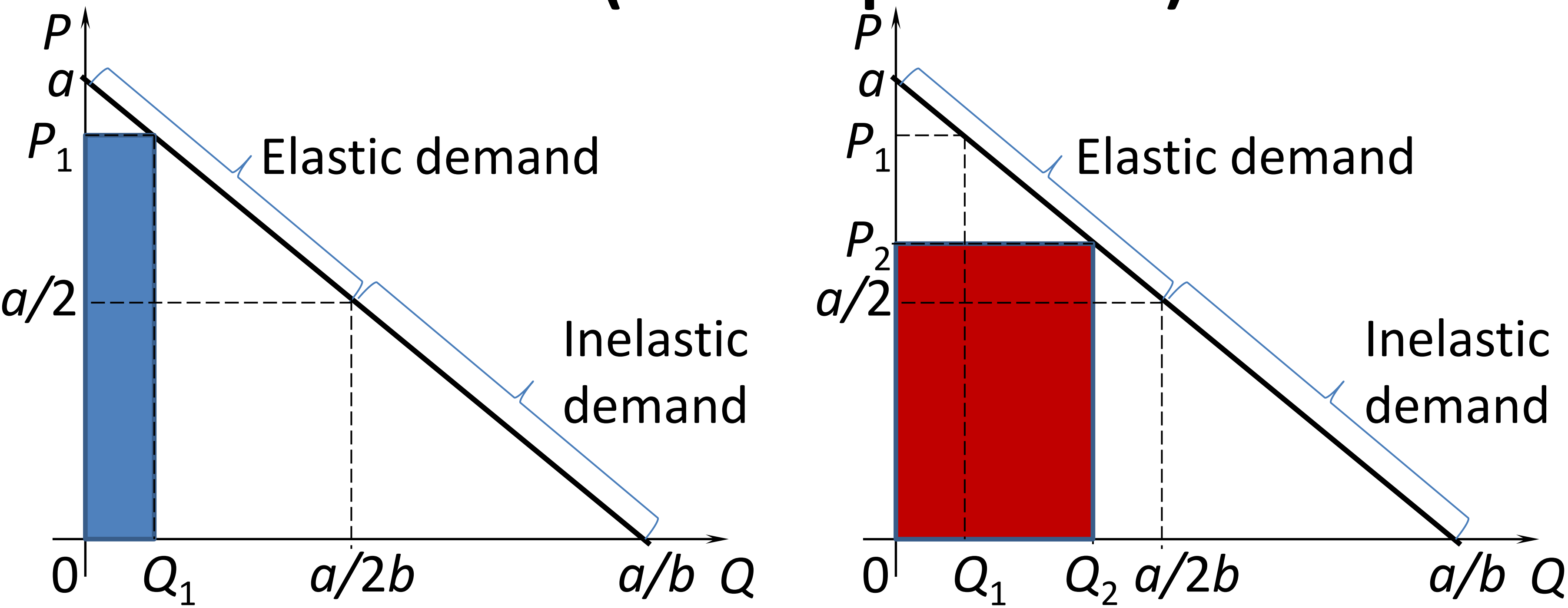
Price Elasticity of Linear Demand

Assume: *P=a-bQ;* i.e. *Q=a/b-P/b*



$$E_p^d = \frac{dQ}{dP} \cdot \frac{P}{Q} = -\frac{1}{b} \cdot \frac{(a-bQ)}{Q} = \frac{bQ-a}{bQ} = 1 - \frac{a}{bQ}$$

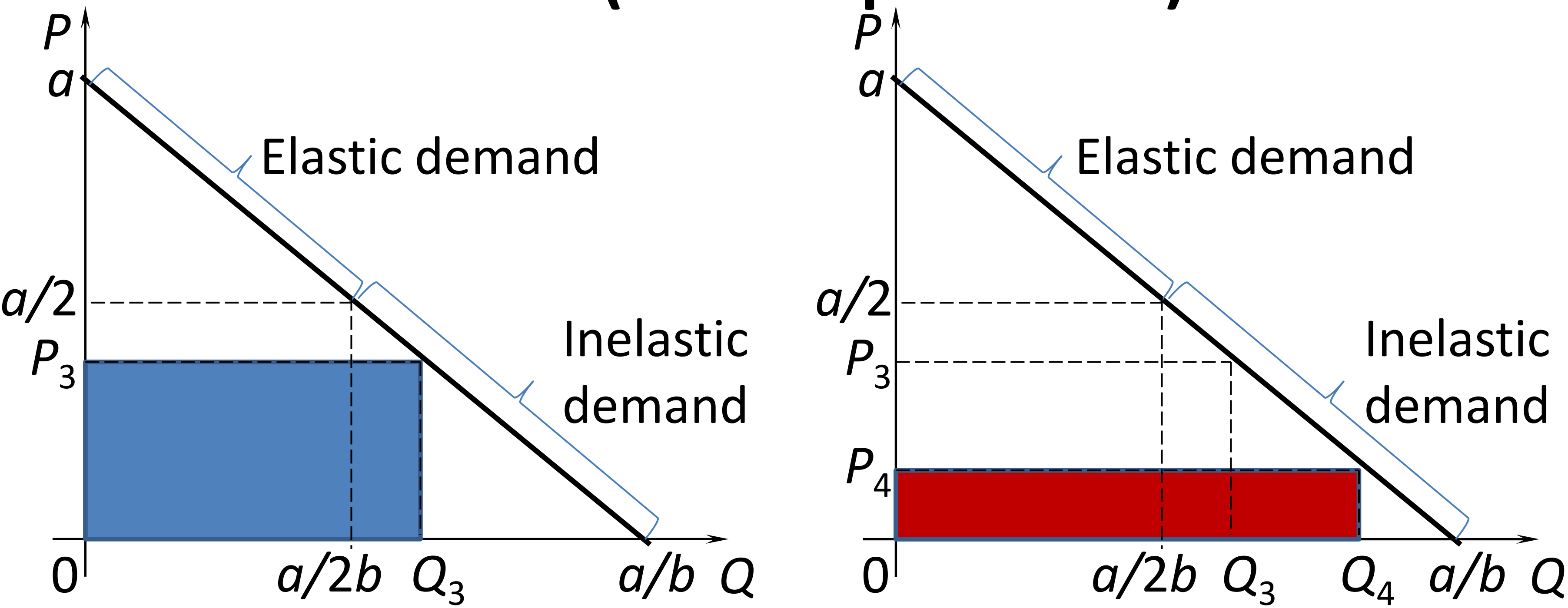
Price Elasticity of Demand and Total Revenue (Total Expenditure)



Total Revenue = Total Expenditure = P(Q)·Q

Elastic demand: TR goes up with an increase in Q and a decrease in P

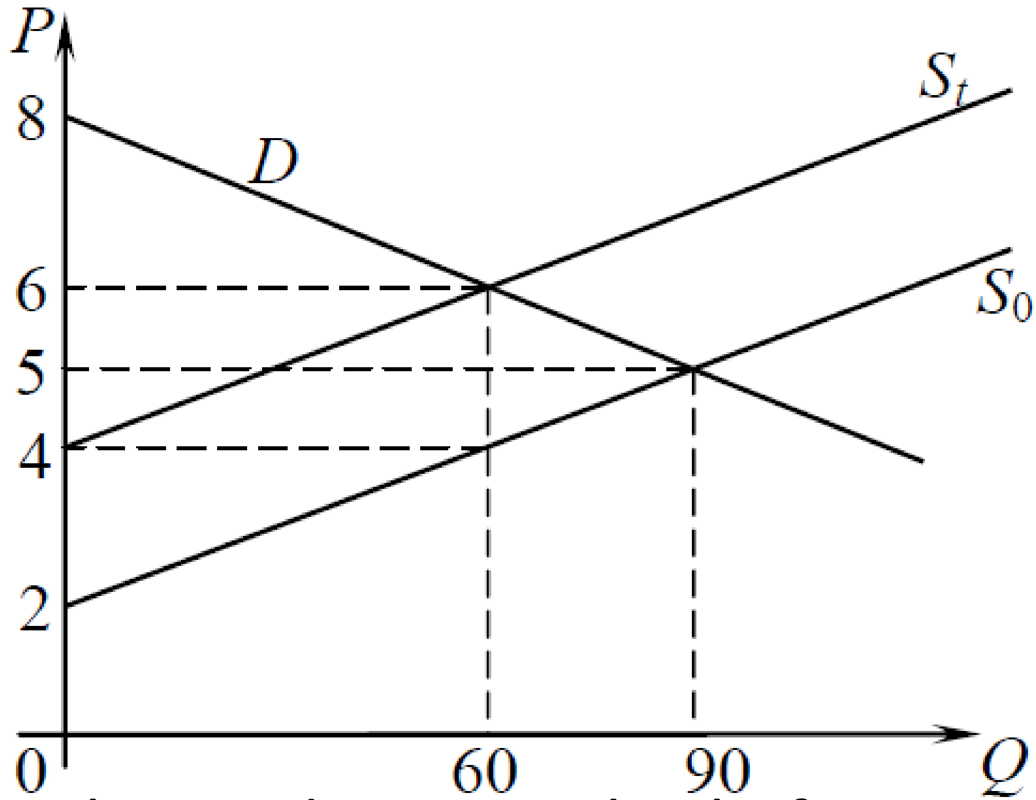
Price Elasticity of Demand and Total Revenue (Total Expenditure)



Total Revenue = Total Expenditure = $P(Q) \cdot Q$

Inelastic demand: TR goes down with an increase in Q and a decrease in P

Unit tax: example (APT 2009)



(a) Calculate the producer surplus before tax.

(b) Now assume a per-unit tax of $2 is imposed whose impact is shown in the graph above.

i. Calculate the amount of tax revenue

ii. What is the after-tax price that the sellers now keep?

iii. Calculate the producer surplus after tax.

(c) Is the demand elastic, inelastic, or unit elastic between the prices of $5 and $6. Explain.

Total Revenue and Marginal Revenue

Total Revenue = Total Expenditure = P(Q) ·Q

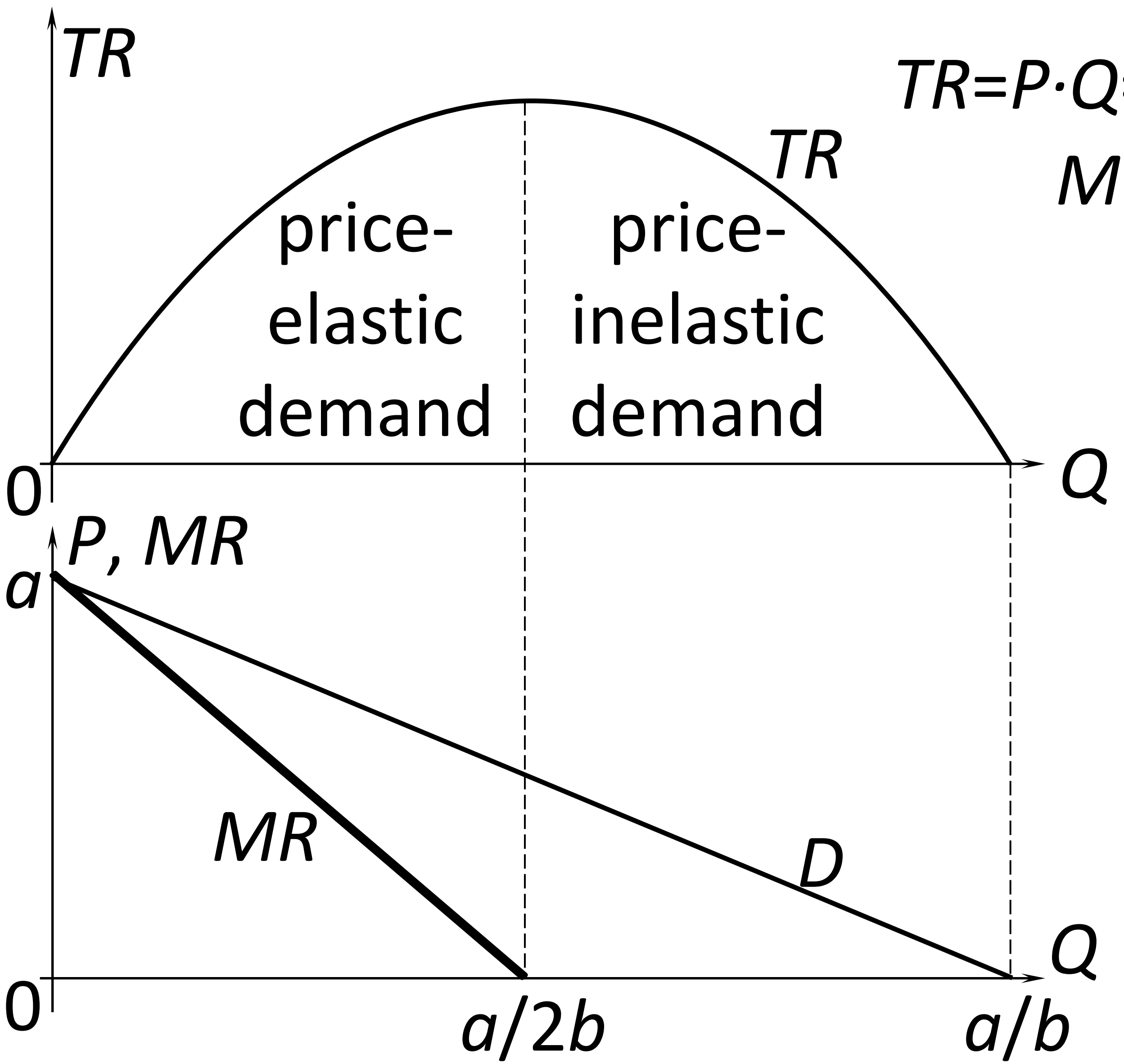
Marginal Revenue: $MR = \frac{\Delta TR}{\Delta Q}$

Marginal Revenue with infinitesimal changes in quantity of the good:

$$MR = \lim_{\Delta Q \to 0} \frac{\Delta TR}{\Delta Q} = \frac{dTR}{dQ}$$

Linear Demand, Total Revenue and Marginal Revenue

$$P = a - bQ$$
$$TR = P \cdot Q = (a - bQ)Q = aQ - bQ^2$$
$$MR = TR' = a - 2bQ$$



Price Elasticity of Demand, Total and Marginal Revenue

$$\frac{dTR}{dQ} = \frac{d}{dQ}(p(Q)Q) = \frac{dp(Q)}{dQ}Q + p(Q) = p(Q)\cdot\left(\frac{dp(Q)}{dQ}\cdot\frac{Q}{p(Q)} + 1\right)$$

$$= p\cdot\left(\frac{1}{\frac{dQ(p)}{dp}\cdot\frac{p}{Q(p)}} + 1\right) = p\cdot\left(1 + \frac{1}{E_p^d}\right).$$

If demand is elastic ($E_p^d < -1$), *MR* is positive: $\frac{dTR}{dQ} > 0$

Total revenue is an increasing function of quantity of the good: when Q goes up, TR grows as well; when Q goes down, TR also declines.

Price Elasticity of Demand, Total and Marginal Revenue

$$\frac{dTR}{dQ} = \frac{d}{dQ}(p(Q)Q) = \frac{dp(Q)}{dQ}Q + p(Q) = p(Q)\cdot\left(\frac{dp(Q)}{dQ}\cdot\frac{Q}{p(Q)} + 1\right)$$
$$= p\cdot\left(\frac{1}{\frac{dQ(p)}{dp}\cdot\frac{p}{Q(p)}} + 1\right) = p\cdot\left(1 + \frac{1}{E_p^d}\right).$$

If demand is inelastic ($-1 < E_p^d < 0$), *MR* is negative: $\frac{dTR}{dQ} < 0$

Total revenue is a decreasing function of quantity of the good:

when Q goes up, TR declines; when Q goes down, TR grows.

Price Elasticity of Demand, Total and Marginal Revenue

$$\frac{dTR}{dQ} = \frac{d}{dQ}(p(Q)Q) = \frac{dp(Q)}{dQ}Q + p(Q) = p(Q)\cdot\left(\frac{dp(Q)}{dQ}\cdot\frac{Q}{p(Q)} + 1\right)$$

$$= p\cdot\left(\frac{1}{\frac{dQ(p)}{dp}\cdot\frac{p}{Q(p)}} + 1\right) = p\cdot\left(1 + \frac{1}{E_p^d}\right).$$

If demand is unit-elastic ($E_p^d = -1$), *MR* is zero: $\frac{dTR}{dQ} = 0$

Total revenue is at the maximum.

Price Elasticity of Demand and Total Expenditure

Total expenditure is the highest when $E_p^d = -1$

If demand is	A decrease in quantity demanded will	An increase in quantity demanded will
Elastic: $E_p^d < -1$	reduce total expenditure	increase total expenditure
Inelastic: $-1 < E_p^d < 0$	increase total expenditure	reduce total expenditure

Cross-Price Elasticity of Demand

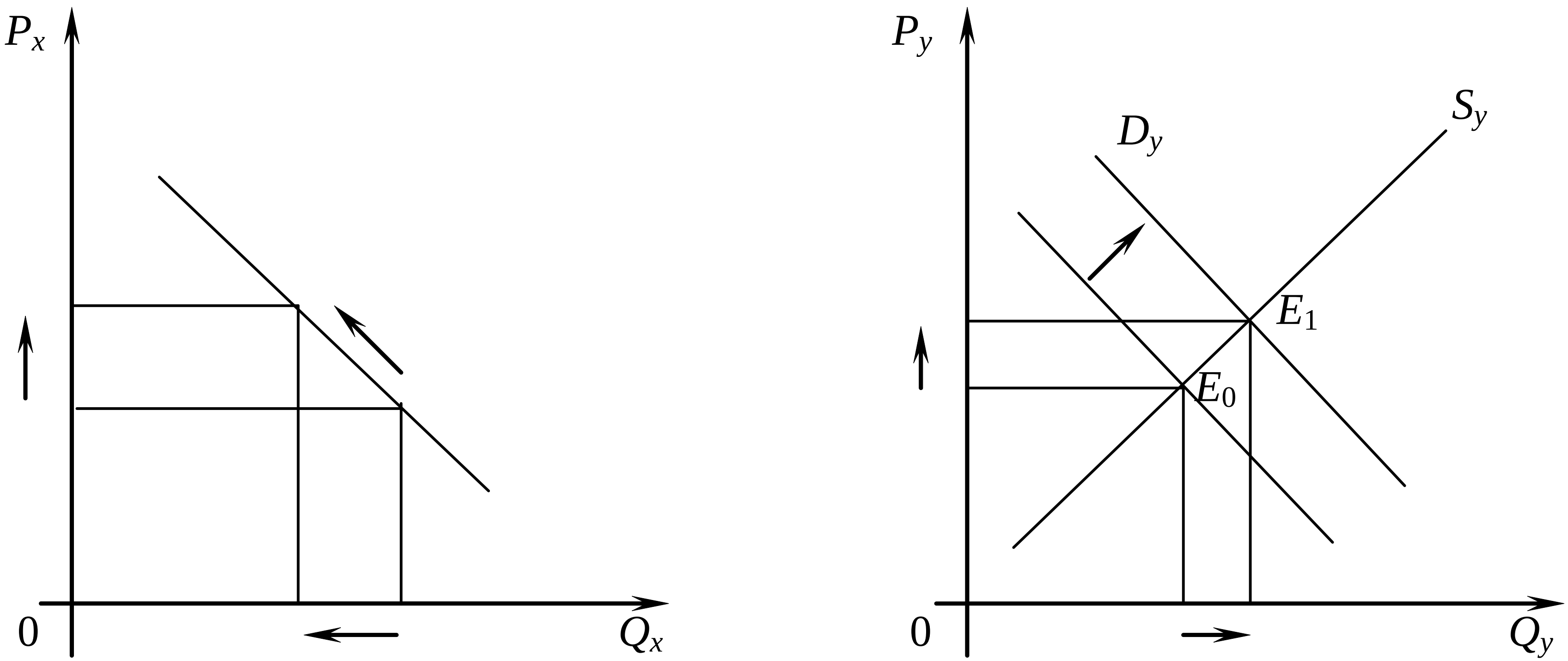
Cross-price elasticity of demand – the percentage of change in quantity of the first good demanded that occurs in response to a 1 percent change in the price of the second one.

$$E_{p_y}^{d_x} = \frac{(Q_2^x - Q_1^x)}{Q_1^x} \Bigg/ \frac{(P_2^y - P_1^y)}{P_1^y} = \frac{\Delta Q_x}{Q_x} \Bigg/ \frac{\Delta P_y}{P_y} = \frac{\Delta Q_x}{\Delta P_y} \cdot \frac{P_y}{Q_x}$$

Cross-Price Elasticity of Demand

Substitutes: cross-price elasticity of demand is positive

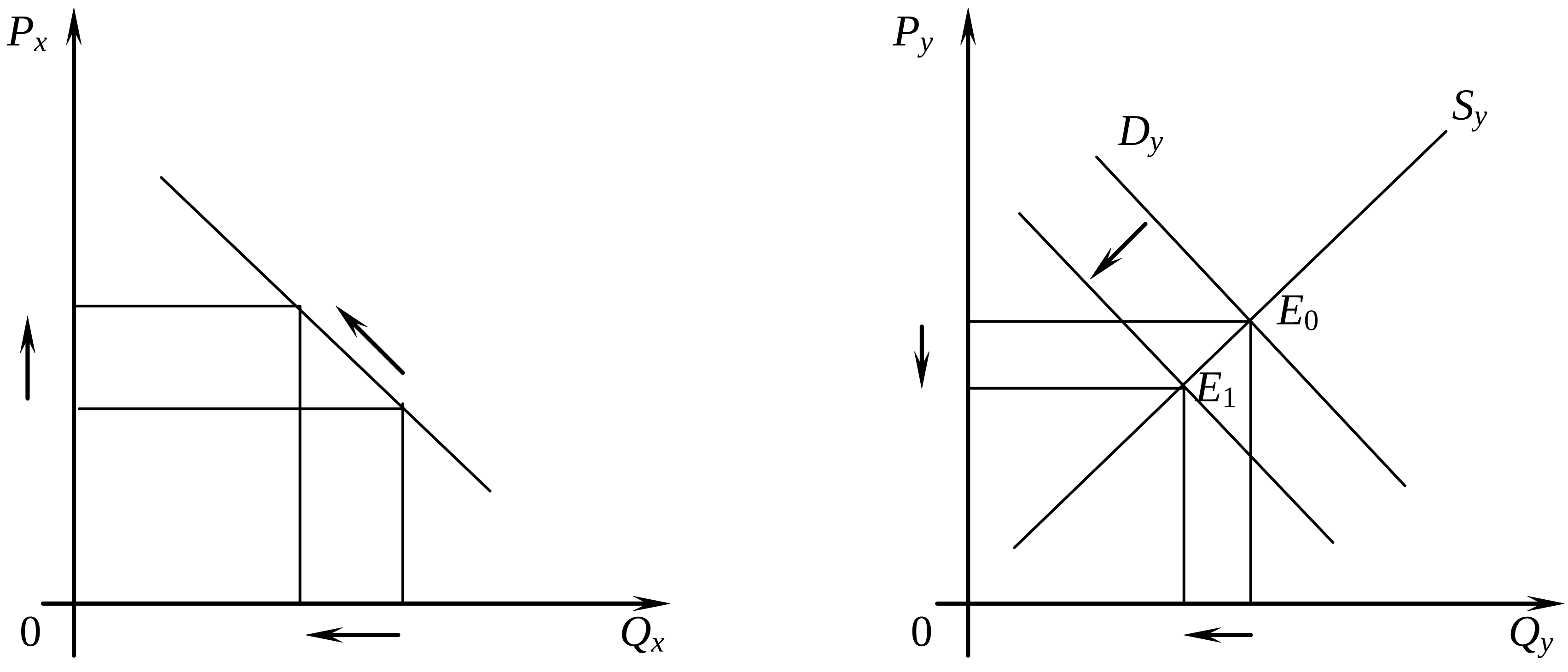
Markets for substitutes



Cross-Price Elasticity of Demand

Complements: cross-price elasticity of demand is negative

Markets for complementary goods



Cross-price elasticity of demand: example (APT 2009)

Assume that the cross-price elasticity of demand between peanuts and bananas is positive. A widespread decease has destroyed the banana crop. What will happen to the equilibrium price and quantity of peanuts in the short run? Explain.

Income Elasticity of Demand

Income elasticity of demand – the percentage of change in the quantity of the first good demanded that occurs in response to a 1 percent change in income.

$$E_I^d = \frac{(Q_2 - Q_1)}{Q_1} \bigg/ \frac{(I_2 - I_1)}{I_1} = \frac{\Delta Q}{Q} \bigg/ \frac{\Delta I}{I} = \frac{\Delta Q}{\Delta I} \cdot \frac{I}{Q}$$

Normal goods: $E_I^d > 0$

Inferior goods: $E_I^d < 0$

Income elasticity of demand: example (APT 2010)

Assume that the income elasticity of demand for good Y is -2. Using a correctly labeled graph of the market for good Y, show the effect of a significant increase in income on the equilibrium price of good Y in the short run.

Price Elasticity of Supply

Price elasticity of supply - the percentage of change in quantity supplied that occurs in response to a 1 percent change in price.

$$E_p^s = \frac{(Q_2 - Q_1)}{Q_1} \bigg/ \frac{(P_2 - P_1)}{P_1} = \frac{\Delta Q}{Q} \bigg/ \frac{\Delta P}{P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

Determinants of Price Elasticity of Supply:

- Production technology and possibility of substitution of inputs;
- Flexibility and mobility of inputs;
- Time horizon: short-run vs. long-run.

Price elasticity of demand and supply : example (APT 2010)

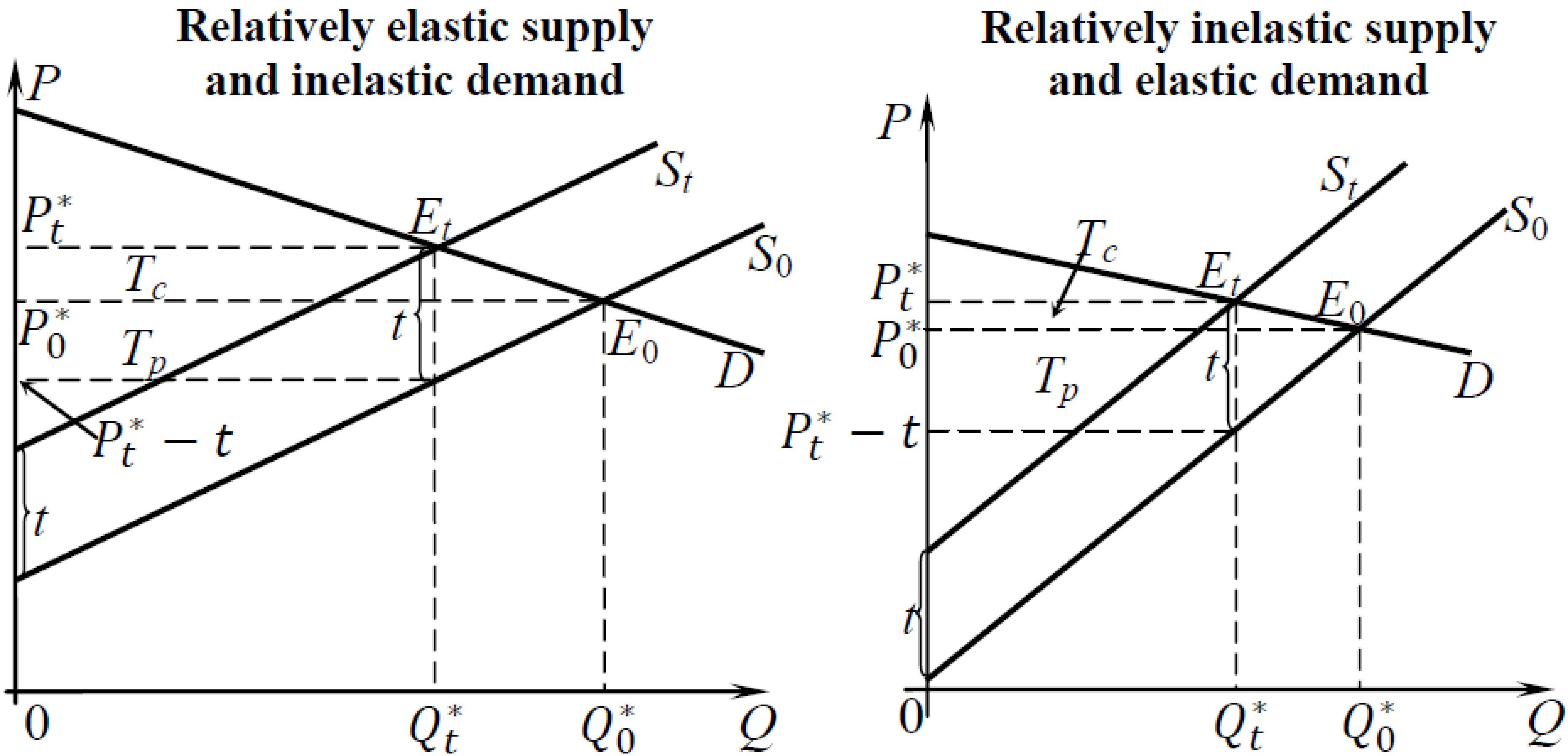
The table below gives the quantity of good X demanded and supplied at various prices.

Price (dollars)	Quantity Demanded (units)	Quantity Supplied (units)
30	1	3
20	3	3
10	4	3

(i) Is the demand for good X relatively elastic, relatively inelastic, unit elastic, perfectly elastic, or perfectly inelastic when the price decreases from $30 to $20? Explain.

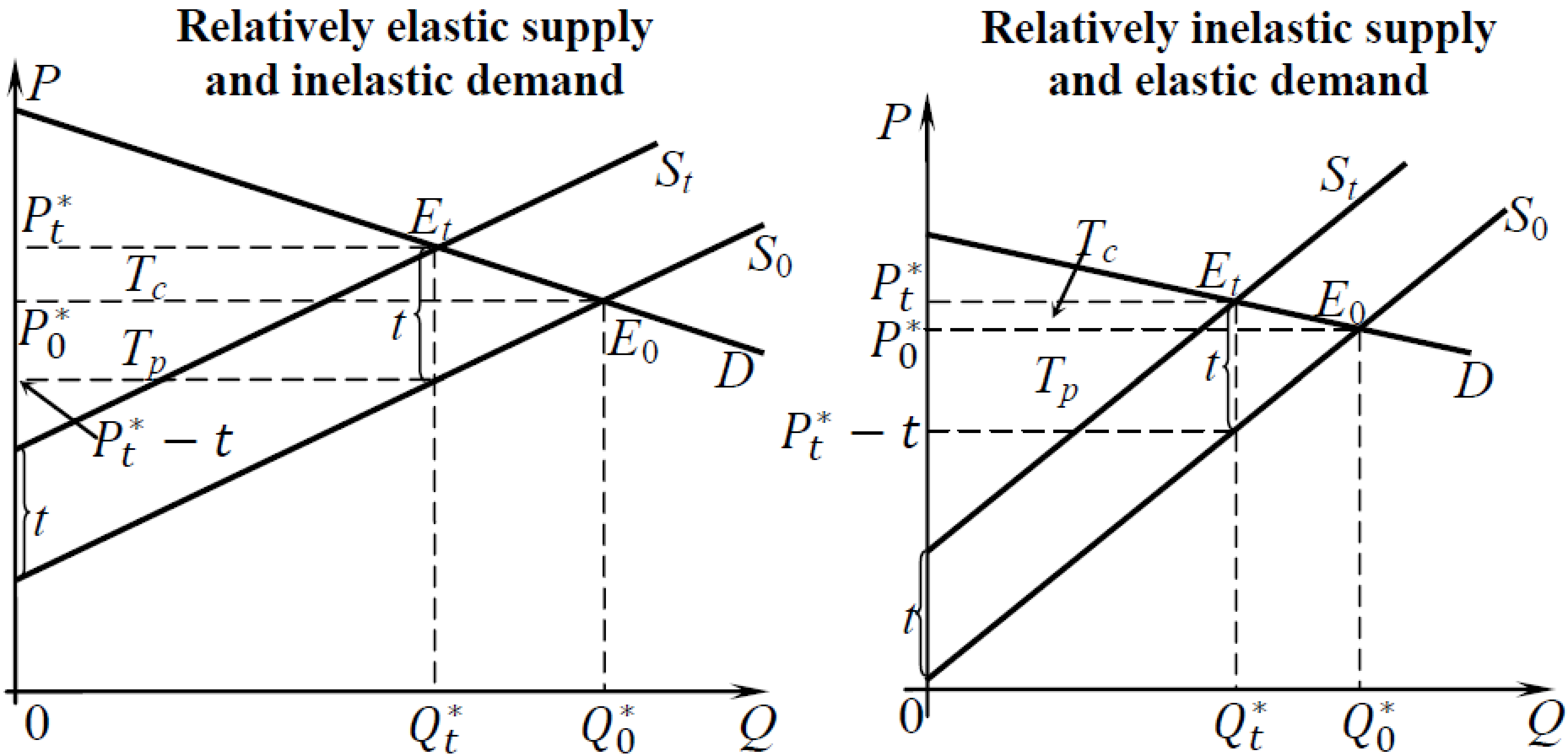
(ii) Is the supply of good X relatively elastic, relatively inelastic, unit elastic, perfectly elastic, or perfectly inelastic when the price decreases from $30 to $20? Explain.

Tax incidence and elasticity of supply and demand



Incidence of a tax describes who eventually bears the burden of it.

Tax incidence and elasticity of supply and demand

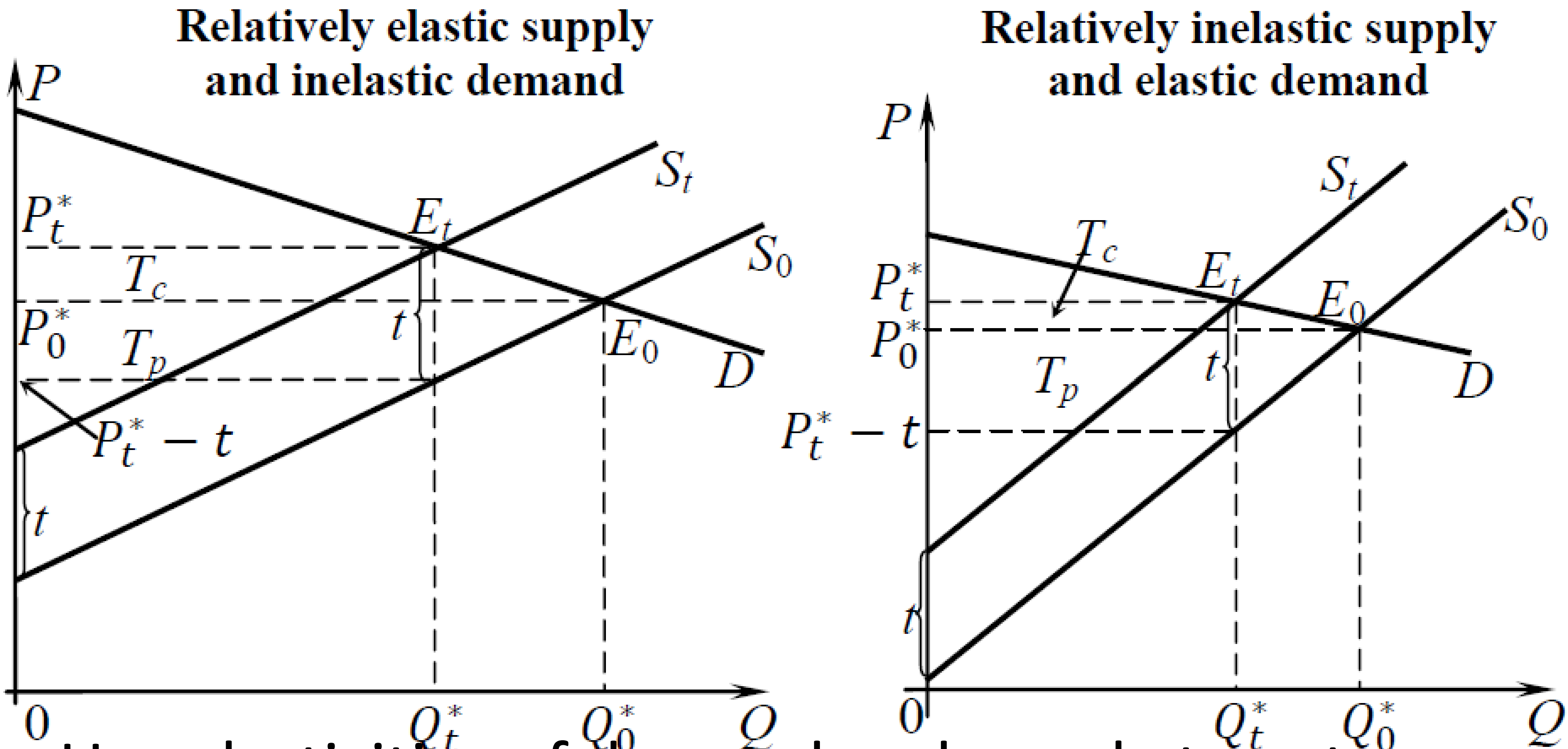


Tax burden of consumers (T_c) and producers (T_p):

$$T = tQ_t^* = T_c + T_p, \quad T_c = Q_t^*(P_t^* - P_0^*), \quad T_p = Q_t^*(P_0^* - P_t^* + t),$$

where T is total tax revenue of the government

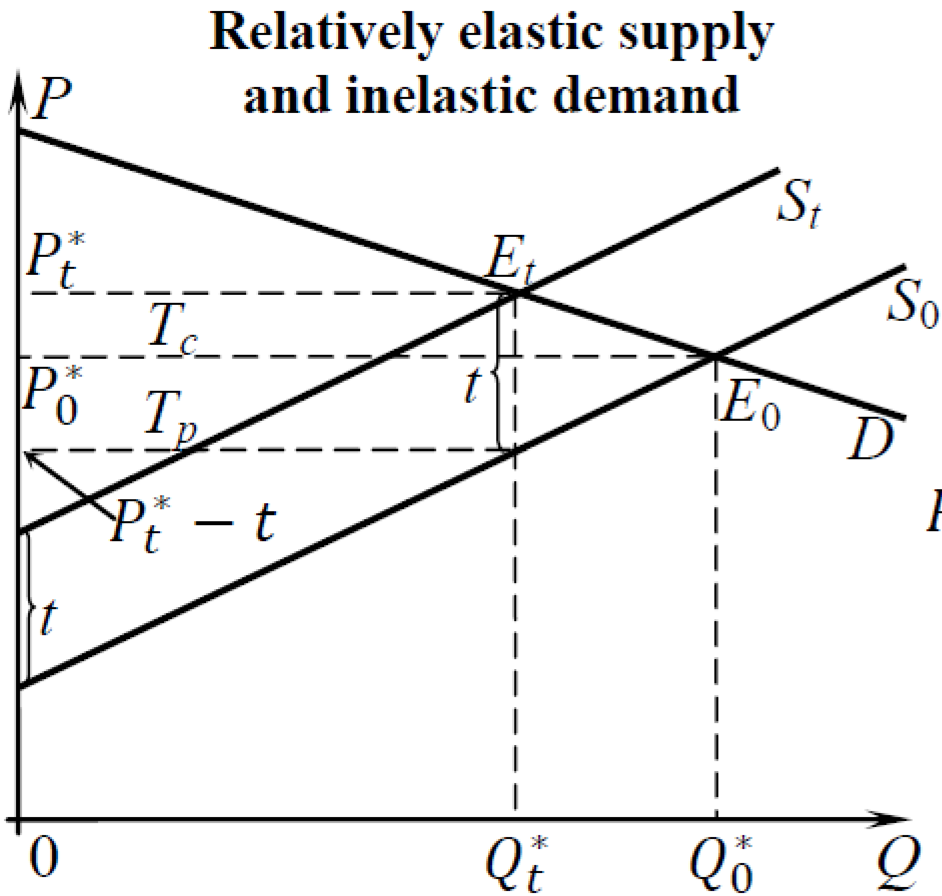
Tax incidence and elasticity of supply and demand

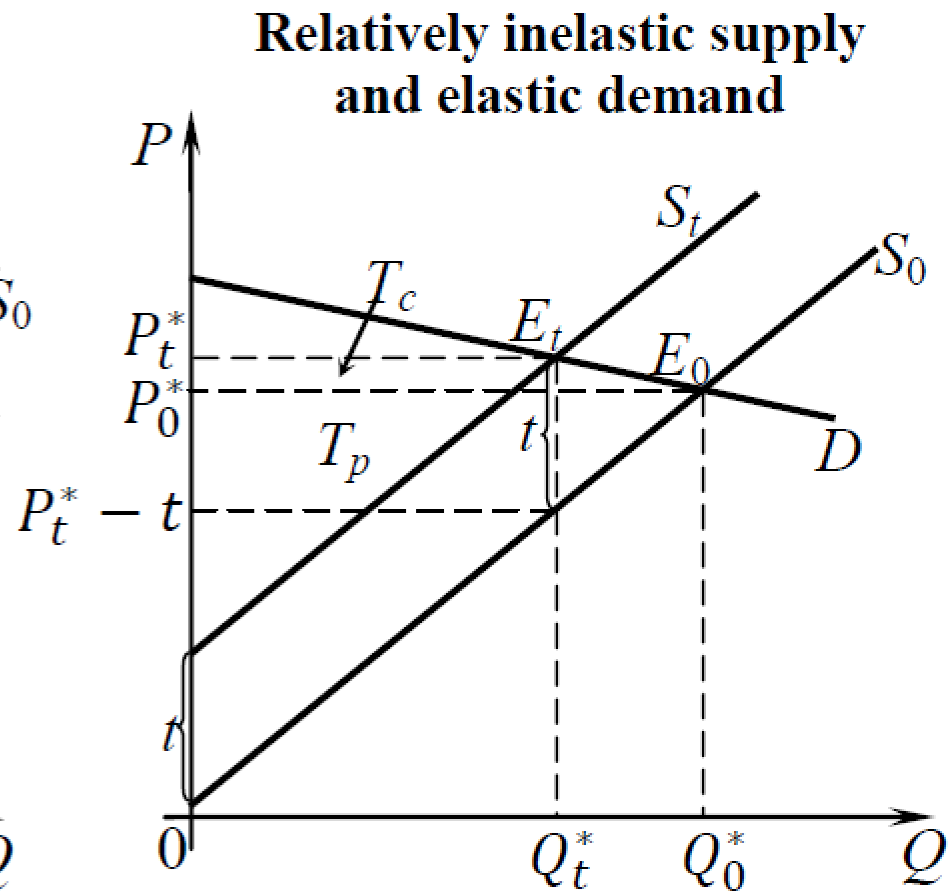


Use elasticities of demand and supply to get:

$$\frac{T_c}{T_p} = \frac{Q_t^*(P_t^* - P_0^*)}{Q_t^*(P_0^* - P_t^* + t)} = -\frac{\dfrac{(Q_t^* - Q_0^*)/Q_0^*}{(P_t^* - t - P_0^*)/P_0^*}}{\dfrac{(Q_t^* - Q_0^*)/Q_0^*}{(P_t^* - P_0^*)/P_0^*}} = -\frac{E_p^s}{E_p^d}$$

Tax incidence and elasticity of supply and demand

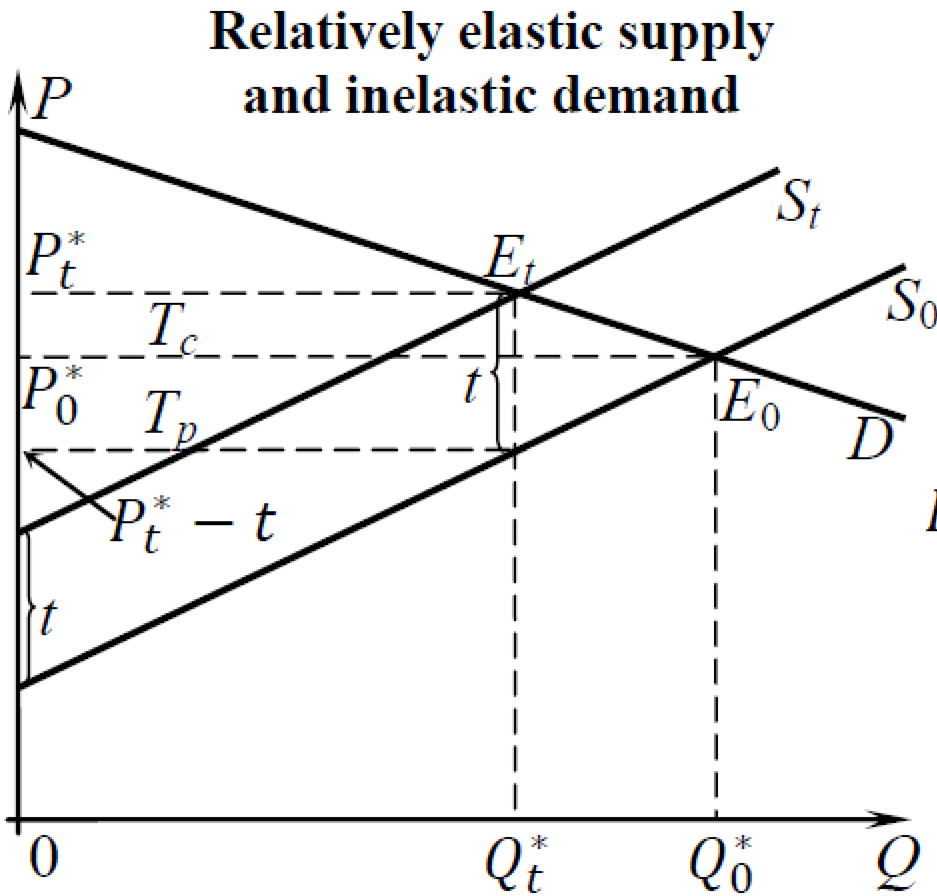


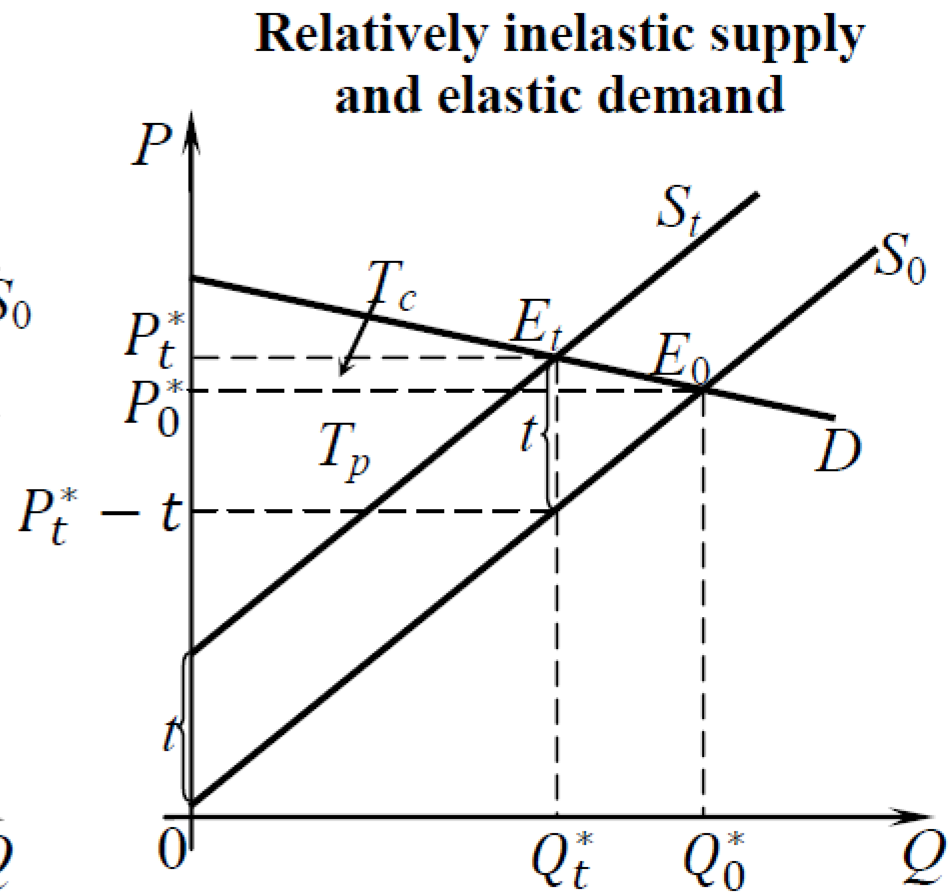


Relative tax burden of consumers and producers is the inverse ratio of absolute values of corresponding elasticities, i.e. the negative of the ratio of elasticities of supply and demand:

$$\frac{T_c}{T_p} = -\frac{E_s}{E_d}$$

Tax incidence and elasticity of supply and demand





The more elastic demand is and the less elastic supply is the greater is the share of the tax levied on producers as compared to that of consumers: $\frac{T_c}{T_p} = -\frac{E_s}{E_d}$

Tax incidence and price elasticity of demand and supply : example (APT 2010)

The table below gives the quantity of good X demanded and supplied at various prices.

Price (dollars)	Quantity Demanded (units)	Quantity Supplied (units)
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(i) Is the demand for good X relatively elastic, relatively inelastic, unit elastic, perfectly elastic, or perfectly inelastic when the price decreases from $30 to $20? Explain.

(ii) Is the supply of good X relatively elastic, relatively inelastic, unit elastic, perfectly elastic, or perfectly inelastic when the price decreases from $30 to $20? Explain.

(iii)If a per-unit tax is imposed on good X, how is the burden of the tax distributed between the buyers and sellers of good X?

Tax incidence and price elasticity of demand: example (APT 2008)

Assume that consumers always buy 20 units of good R each month regardless of its price.

(i) What is the numerical value of the price elasticity of demand for good R?

(ii) If the government implements a per-unit tax of $2 on good R, how much of the tax will the seller pay?