

## Complicated alternatives to majority rule

In this Method [the Method of Marks], a certain number of marks is fixed, which each elector shall have at his disposal; he may assign them all to one candidate, or divide them among several candidates, in proportion to their eligibility; and the candidate who gets the greatest total of marks is the winner.

This method would, I think, be absolutely perfect, if only each elector wished to do all in his power to secure the election of *that candidate who should be the most generally acceptable*, even if that candidate should *not* be the one of his own choice: in this case he would be careful to make the marks exactly represent his estimate of the relative eligibility of *all* the candidates, even of those he *least* desired to see elected; and the desired result would be secured.

But we are not sufficiently unselfish and public-spirited to give any hope of this result being attained. Each elector would feel that it was *possible* for each other elector to assign the entire number of marks to his favorite candidate, giving to all the other candidates zero: and he would conclude that, in order to give his *own* favorite candidate any chance of success, he must do the same for him.

Charles Dodgson (Lewis Carroll)

In 1954, in what has become the classic paper on public goods, Paul Samuelson both defined the necessary conditions for Pareto optimality in the presence of public goods and cast a pall over the field of public economics by asserting that no procedure could be constructed to reveal the information on preferences required to determine the quantities of public goods that would satisfy the Pareto-optimality condition. In a section entitled “Impossibility of Decentralized Spontaneous Solution,” Samuelson (1954, p. 182) stated that “*no decentralized pricing system can serve to determine optimally these levels of collective consumption*” (italics in original).

So influential was this article, that for a generation economists merely repeated Samuelson’s words and lamented the absence of a satisfactory procedure for revealing individual preferences. And with good reason. Traditional voting schemes seemed vulnerable to the transaction costs and strategic incentives inherent in the unanimity rule, or the paucity of information and onus of compulsion characterizing less-than-unanimity rules, most notably the majority rule.

But then in the seventies, a revolution suddenly erupted. New procedures began to appear one after the other, which claimed to have solved the preference revelation problem. As so often happens in the mechanical arts, once one scientist demonstrated that the impossible might be possible, others were moved to follow, and a wave of developments ensued. In this chapter we review this literature, focusing upon three

Table 8.1.

Voter	Issue		Tax
	<i>P</i>	<i>S</i>	
<i>A</i>	30		20
<i>B</i>		40	0
<i>C</i>	20		10
Total	50	40	30

rather different types of procedures. We begin with the procedure that has attracted the greatest attention.

## 8.1      **The demand-revealing process**

### 8.1.1    *The mechanics of the process*

This procedure was first described by Vickrey in 1961, although he attributed the idea to “an interesting suggestion” Lerner threw out in *Economics of Control* (1944). Consequently, the procedure might be said to antedate Samuelson’s paper by 10 years. But neither Lerner nor Vickrey applied the procedure to the problem of revealing preferences for public goods, and its potential importance was not recognized until the appearance of papers by Clarke (1971, 1972) and Groves (1973).

To understand how the procedure works, consider the collective choice between the two issues *P* and *S*. Assume a committee of three with preferences as given in Table 8.1. Voter *A* expects to be the equivalent of \$30 better off from the victory of *P*, voter *C* \$20, and voter *B* prefers *S* by the equivalent of \$40. The procedure for selecting a winner is to first ask all three voters to state in dollars the amount of benefits they expect from the victory of their preferred issue, and then add these figures, declaring the issue with the most expected benefits the winner. In the present example this is *P*, since it promises gains of 50 to voters *A* and *C*, whereas *S* benefits *B* by only 40.

The voters are induced to declare their true preferences for the issues by announcing that they will be charged a certain tax, depending on the responses they make and their impact on the final outcome. This tax is calculated in the following way: the dollar votes of all other voters are added up and the outcome determined. The voter-in-question’s dollar votes are now added in to see if the outcome is changed. If it is not, he pays no tax. If it is, he pays a tax equal to the *net* gains expected from the victory of the other issue in the absence of his vote. Thus, a voter pays a tax only when his vote is decisive in changing the outcome, and then pays not the amount he has declared, but the amount needed to balance the declared benefits of the other voters on the two issues. The last column of Table 8.1 presents the taxes on the three voters. Without *A*, there are 40 dollar votes for *S* and 20 for *P*. *A*’s vote is decisive in determining the outcome, and imposes a net cost of 20 on the other two voters, and that is *A*’s tax. *B*’s vote does not affect the outcome, and he pays no

Table 8.2.

Voter	Issue		Tax
	<i>P</i>	<i>S</i>	
<i>A</i>	30		10
<i>B</i>		40	0
<i>C</i>	20		0
<i>A'</i>	30		10
<i>B'</i>		40	0
<i>C'</i>	20		0
Total	100	80	20

tax. Without *C*'s vote, *S* would again win, so *C* pays a tax equal to the net benefits the other voters would have received had he not voted ( $40 - 30 = 10$ ).

Under the tax each voter has an incentive to reveal his true preferences for the two issues. Any amount of benefits from *P* that voter *A* declared equal to or greater than 21 would leave the collective decision, *and his tax*, unchanged. If he declared net benefits of less than 20, *S* would win, and *A*'s tax would fall from 20 to 0, but his benefits of 30 would also disappear. A voter pays a tax only if his vote is decisive, and the tax he pays is always equal to or less than the benefits he receives. Thus, there is no incentive to understate one's gains, for then one risks foregoing a chance to cast the deciding vote at a cost less than the benefits. And there is no incentive to overstate one's preferences, since this incurs the risk of casting the decisive vote and receiving a tax above one's actual benefits, albeit less than one's declared benefits. The optimal strategy is honest revelation of preferences.

To maintain this desirable incentive property, the tax revenue raised to induce honest revelation of preferences cannot be returned to the voters in such a way as to affect their voting decision. The safest thing to do with the money to avoid distorting incentives is to waste it. But this implies that the outcome from the procedure will not be Pareto optimal (Groves and Ledyard, 1977a,b; Loeb, 1977). The amount by which the procedure falls short of Pareto optimality can be stated explicitly: it is the amount of revenue raised by the incentive tax. In the example above, this amount is substantial, equaling three times the net gains from collective action.

Fortunately, the amount of taxes raised under the demand-revealing procedure should decline as the number of voters increases (Tideman and Tullock, 1976, 1977). To see why this is so, consider Table 8.2, in which the preferences of three other voters, *A'*, *B'*, and *C'*, identical to those of *A*, *B*, and *C*, have been included. The issue *P* still wins, of course, now by a surplus of 20. Voter *C*'s tax has fallen from 10 to 0, however, and *A*'s from 20 to 10. Without voter *C*, the net benefits on the two issues over the other voters are 0 (80 for *P* and 80 for *S*). Although his vote tips the outcome in favor of *P*, his gain of 20 does not come at the *net* expense of the other voters. So *C* pays no tax. *A* still pays a positive tax, but the amount has been reduced, since the net cost of his vote on all other voters has fallen. With the addition of three more voters (*A''*, *B''*, *C''*) with preferences identical to *A*, *B*, and *C*, the outcome would again not change, and the taxes on all voters would now be zero.

Thus, the collective decision of this committee of nine would be Pareto optimal. Although the procedure does allow for a weighing of intensities in determining the outcome, the effect of any one voter's preferences on the final outcome will dwindle, as with other voting procedures, as the number of voters increases. Since a voter's tax equals his impact on the other voters, it too dwindles as the size of the group increases.

Groves and Ledyard (1977c, p. 140) claim to be able to construct counterexamples in which the incentive tax surplus is arbitrarily large, and Kormendi (1979, 1980) has pressed the same point. But such examples rely on expanding the committee by adding equal numbers of voters who favor *P* and *S*. If the committee were equally divided between voters favoring *P* and voters favoring *S*, every vote might be decisive and the amount of tax revenue raised would be large, whereas the *net* social benefit would be very small. However, we would then have essentially a distributional issue, the *P*s versus the *S*s. For a pure public good that all favor, the incentive-tax revenue should vanish as *n* increases. For a rigorous demonstration, see Rob (1982).

The procedure can reveal individual demand schedules for a public good, from whence its name arises. We follow here the exposition of Tideman and Tullock (1976). Each individual is asked to report his complete demand schedule for the public good. These schedules are then vertically added to obtain the aggregate demand for the public good. The intersection of this schedule and the supply schedule for the good determines the quantity provided. If each individual has honestly reported his demand schedule, the procedure determines the Pareto-optimal quantity of public good, as defined by Samuelson (1954) and Bowen (1943).

Individuals are again induced to reveal their true preferences via a special tax imposed upon them. In fact, there are two taxes imposed upon the individual, one designed to cover the full costs of producing the public good and the other to ensure honest revelation of preferences. In our first example, the first of these two taxes was implicitly assumed to be part of the proposals *P* and *S*. Let us assume that the public good can be supplied at constant unit costs *C*, and that each voter is assigned a share of these costs,  $T_j$ , such that  $\sum_{j=1}^n T_j = C$ . These  $T_j$ s are the first components of each individual's tax. The other component is computed in a way analogous to that used to assign each individual a tax in the preceding example. Namely, one first determines the quantity of public good that would be demanded in the absence of individual *i*'s demand schedule and contribution to the public good's total costs. The quantity with his demand schedule and contribution is determined next. The difference represents the impact of this individual's preferences on the collective outcome. The cost to the other voters of the shift in quantity that recording his preferences brings about is the absolute value of the difference between the costs of producing these extra units and the sum of the individual demand schedules over these units. Thus, if *i* forces the community to consume more than it would have without his demand-schedule vote, the costs of the extra output will exceed their willingness to pay for it, and *i* is charged the difference. Conversely, if voter *i* causes the community to consume less than they would have, their aggregate demand for the extra units of public good will exceed the good's costs, and the

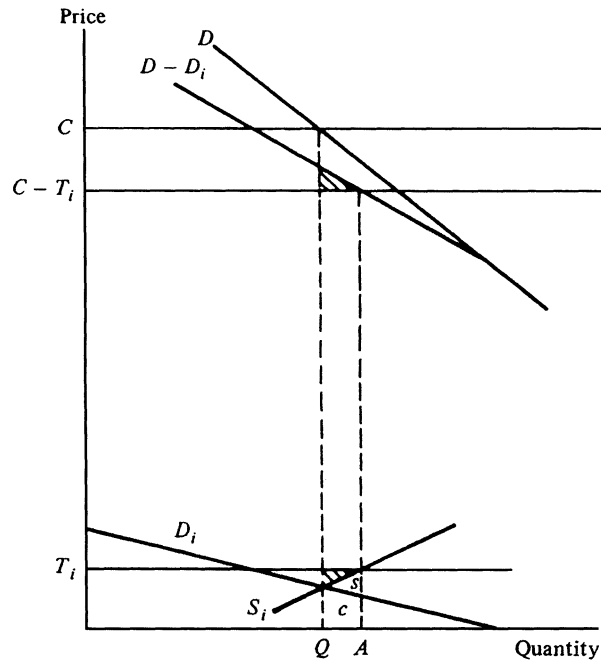


Figure 8.1. Some new processes for revealing preferences.

difference, the loss in consumers' surplus to the other voters, is charged to the  $i$ th voter.

The latter possibility is illustrated with the help of Figure 8.1. Omitting  $i$ 's demand schedule, aggregate demand for the public good is  $D - D_i$ . Subtracting his preassigned tax share, the cost of the public good is  $C - T_i$ . With  $i$ 's preferences removed, the community would purchase  $A$ . With  $i$ 's preferences included, the community purchases  $Q$ , the quantity at which aggregate demand and supply are equal. The cost imposed on the other voters of this shift in outcomes is the difference between the amount that the other voters would be willing to pay for the extra units ( $A - Q$ ) and the taxes they would have to pay  $(C - T_i)(A - Q)$  for these units, which is the cross-hatched triangle above the line  $C - T_i$ . This triangle represents the additional tax, apart from  $T_i Q$ , that the  $i$ th voter must pay.

That the  $i$ th voter's optimal strategy is to reveal his true demand schedule in the presence of this incentive tax becomes clear when we construct an effective supply schedule of the public good,  $S_i$ , to the  $i$ th voter, by subtracting the  $D - D_i$  schedule from  $C$ . The intersection of the individual voter's demand for the public good,  $D_i$ , and this  $S_i$  schedule is for him the optimal quantity of public good, which, of course, is  $Q$ . By stating his demand schedule as  $D_i$ , voter  $i$  forces the community to consume  $Q$  instead of  $A$ , and thereby saves himself the rectangle  $T_i(QA)$  in taxes. He must pay the incentive tax represented by the cross-hatched triangle below  $T_i$ , which equals the cross-hatched triangle above  $C - T_i$ , and loses the consumers' surplus represented by the quadrilateral,  $c$ . His net gain from forcing the community to  $Q$  rather than leaving it at  $A$  is thus the triangle  $s$ . That there is nothing to be gained

by stating a demand schedule below  $D_i$  can be seen by observing that the triangle  $s$  vanishes at  $Q$ . To the left of  $Q$ ,  $i$ 's incentive tax plus consumer surplus loss would exceed his tax saving  $T_i$ . If he states any demand schedule above  $D_i$ ,  $T_i$  exceeds his consumer surplus gain and incentive tax saving. The honest revelation of his true demand schedule  $D_i$  is  $i$ 's optimal strategy.

To see how the procedure works algebraically, write  $U_i(G)$  as voter  $i$ 's utility from consuming  $G$ . Let  $t_i$  be  $i$ 's incentive tax. We ignore income effects, so we can assume that the marginal utility of money is constant and measure  $U_i(G)$  in dollar units. Voter  $i$ 's objective is thus to maximize utility,  $U_i$ , net of  $i$ 's share of the cost of the public good,  $T_i G$ , and incentive tax,  $t_i$ ; that is,

$$O_i = U_i(G) - T_i G - t_i. \quad (8.1)$$

The incentive tax that  $i$  must pay is the cost that  $i$ 's vote imposes on all other voters by bringing about  $G$ ; it is the difference between the other voters' utilities at  $G$  and their cost shares:

$$t_i = \sum_{j \neq i} (T_j G - U_j(G)). \quad (8.2)$$

Substituting (8.2) into (8.1) and maximizing with respect to  $G$ , one obtains

$$dO_i/dG = U_i'(G) - T_i - \sum_{j \neq i} (T_j - U_j'(G)). \quad (8.3)$$

Setting (8.3) equal to zero, we can solve for the optimal  $G$  for  $i$  to state, given  $i$ 's tax share  $T_i$  and the incentive tax  $t_i$ . Rearranging this first-order condition, we obtain the Samuelsonian condition for the Pareto-optimal provision of  $G$ :

$$\sum_i U_i'(G) = \sum_i T_i = C. \quad (8.4)$$

Note that although the quantity of the public good selected is Pareto optimal, it is also generally true that  $U_i'(G) \neq T_i$ ,  $i = 1, n$ , as can also be seen in Figure 8.1. An important element of the procedure is that an individual's share of the cost of a public good is independent of his stated demand schedule. This independence is necessary to ensure the honest revelation of preferences. Only the (probably rather small) incentive tax, represented by the cross-hatched triangle in Figure 8.1, is directly related to the individual's reported demand schedule, and the funds raised here are to be wasted, or at least not returned in any systematic way to the payer.

The idea of a two-part tariff to ensure an efficient allocation of resources in industries characterized by economies of scale, or large fixed costs, has been around for some time. The most obvious examples are probably the electric and gas industries (see, e.g., Kahn, 1970, pp. 95–100). The principles underlying these pricing schemes are analogous to those of the demand-revealing process. A proportional charge is made to each customer for his use of the service, and an extra charge is made for the costs on other buyers that a customer's demand imposes at the peak (margin) of the system's capacity. Public goods are also characterized by high fixed

costs, the joint supply property; and the demand-revealing process is thus a perhaps not-too-surprising, if somewhat long-awaited, extension of the idea of the two-part tariff into the public good area.

Green and Laffont (1977a) have demonstrated that the class of demand-revealing processes first developed by Groves (1973) in effect defines the full set of procedures of this type, of which the preceding examples concern but one variant, for which honest revelation of preferences is the dominant strategy. That is, regardless of what message the other voters supply to the message-gathering agent, it is always an individual's optimal strategy to reveal his true preferences. This property of the procedure is dependent on an absence of interaction between an individual's fixed tax share, revealed demand schedule, and the revealed demand schedules of the other individuals. There is no way, direct or indirect, by which individuals can influence the taxes that they pay other than through the immediate effect of their revealed demand schedules. Thus, the procedure is a purely partial equilibrium approach that abstracts from any interactions among voters via income effects or other means.

Although honest preference revelations and the Samuelson efficiency conditions are ensured under the partial equilibrium variants of the demand-revealing process, budget balance is not, and so Pareto efficiency cannot be presumed. As already noted, the size of the total tax intake from the incentive tax is a matter of some controversy, and so, too, therefore the significance of the Pareto-inefficiency property. Groves and Ledyard (1977a) developed a general equilibrium version of the demand-revealing process in which budget balance is achieved. Each individual reports a quadratic approximation to his true demand function of the following form:

$$m_i = \beta_i G - \frac{\gamma}{2n} G^2, \quad (8.5)$$

where  $\gamma$  is a constant across all individuals,  $G$  is the quantity of public goods, and  $n$  is the number of consumers. The individual's tax is given as

$$T_i = a_i G^*(m) + \frac{\gamma}{2} \left[ \left( \frac{n-1}{n} \right) (m_i - \mu_i)^2 - \sigma_i^2 \right], \quad (8.6)$$

where  $a_i$  is a preassigned tax share,  $G^*(m)$  is the quantity of public good chosen as a result of the aggregation of all individual messages,  $\mu_i$  is the mean of all of the *other* voters' messages, and  $\sigma_i$  is the standard error of all of the other voters' messages. Each individual pays a fixed tax share,  $a_i$ , and variable tax that increases with the size of the difference between his proposed quantity and the proposed quantity of all other voters, and decreases in proportion to the amount of dispersion among the other proposals. Thus a voter is again penalized to the extent that his proposed public good quantity differs from that of all other voters, but his penalty is smaller, the more disagreement there is among the other voters over the desired quantity of public good. To supply his optimal message, a voter must know his preassigned tax share, the fixed constant, and the mean and standard error of all other voters' messages. Thus, a sequential adjustment procedure is required in which each voter is supplied with the computed mean and standard error of the other voters' messages

on the preceding round of calculations to make a calculation in the present one. The present messages then become the data for making new mean and standard error statistics for each voter. The process continues until equilibrium is obtained.<sup>1</sup>

Under the Groves-Ledyard procedure, the tax on each individual can be designed to ensure budget balance, and if each voter treats the messages of the others as given, each has the incentive to reveal his own preferences honestly, and a Pareto-optimal equilibrium can be established (1977a, pp. 794–806). But it may not be in each voter's best interests to treat the messages of all other voters as given. The achievement of budget balance and individual equilibrium via a multistep adjustment process makes each individual's message at one step of the process dependent on the other individuals' messages at the preceding stage. A voter who could deduce the effect of his message on the messages of other voters in subsequent rounds of voting might have an incentive to manipulate their messages in later rounds via dishonest indication of his own demand schedule in earlier rounds. The proofs of Pareto optimality that Groves and Ledyard offer assume essentially Cournot-type behavior: each voter treats the messages of the other voters as fixed at each stage of the adjustment process. Once voters begin to take the reactions of other voters into account, Stackelberg-type behavior may be individually optimal, and both the honest-revelation and Pareto-efficiency properties of the mechanism may be lost (Groves and Ledyard, 1977b, pp. 118–20; Groves, 1979; Margolis, 1983).

Although honest revelation of individual preferences is not the dominant strategy under the Groves-Ledyard balanced budget variant of the demand-revealing procedure, it is a Nash equilibrium. That is, given that all other individuals honestly reveal their preferences at each step in the process, it is in each voter's best interest to do so. The significance of this property of the procedure rests heavily on whether it is reasonable to expect voters to adopt a Cournot-type frame of mind when sending messages, at least when the number of voters is fairly large. This issue cannot be settled on the basis of *a priori argument*.<sup>2</sup>

Many criticisms were levied against the family of demand-revelation processes when they were first proposed. One set of these concerns the revenue raised by the incentive tax. To preserve the incentive properties of the procedure, the revenue collected through the incentive tax paid by individual  $i$  cannot be returned to her. This problem could easily be circumvented without having to burn the money raised by the incentive tax. If, for example, two communities of roughly equal size were to use the procedure, they might simply agree to swap incentive tax revenue each year and return the funds on a pro rata basis to the citizens. Bailey (1997) proposed giving each person an equal portion of the incentive tax revenues paid by the other  $n - 1$  citizens in the community.

Potentially more serious is the problem raised when the incentive tax revenue is large enough to induce significant income effects. Once income effects are allowed,

<sup>1</sup> Groves and Loeb (1975) first discussed the possibility of achieving budget balance when the consumer's, in this case a firm's, demand schedule is a quadratic function of the form previously given.

<sup>2</sup> The basic result is established by Groves and Ledyard (1977a). For a discussion of its significance, see Greenberg, Mackay, and Tideman (1977), and Groves and Ledyard (1977c).



however, we move into the general equilibrium framework first explored by Groves and Ledyard (1977a). To handle income effects adequately, one needs even stronger assumptions and a more complicated voting procedure than Groves-Ledyard (Conn, 1983),<sup>3</sup> and the dominance property of preference revelation vanishes.<sup>4</sup>

The remaining difficulties of the process are shared by most, if not all, other voting processes:

**Information incentives:** To the extent that the size of the incentive tax levied on any individual falls as the number of voters increases, the incentive to provide information conscientiously dwindles.<sup>5</sup> Thus, the one-step demand-revealing process is caught in a form of numerical dilemma. If the numbers involved are small, the incentive taxes may be large, but then, so too is the potential problem arising from significant income effects. If the numbers are large, the Pareto inefficiency may be relatively small, but so too is the incentive to supply the needed information. Much of the information coming from the process could be inaccurate, although not systematically dishonest. Clarke (1977), Green and Laffont (1977b), Tullock (1977a), and Brubaker (1986) have discussed ways to circumvent this problem by relying on representative systems or sampling techniques.

**Coalitions:** A coalition of voters who felt they would be 100 better off from the victory of  $P$  could increase the chances of  $P$ 's winning significantly by all agreeing to claim that they were 200 better off under  $P$ 's victory. As long as  $P$  won by more than 200, they would be better off under the coalition than acting independently. If  $P$  won by less than 100 or lost, they would be no worse off. Only if  $P$  won by between 100 and 200, an unlikely event if the coalition is very large, would a voter be worse off under the outcome with the coalition than without it. Thus, incentives to form coalitions to manipulate outcomes exist under the demand-revealing process (Bennett and Conn, 1977; Riker, 1979).

Tullock (1977c) is undoubtedly correct in arguing that the problem of coalition formation is unlikely to be serious if the number of voters is large and voting is by secret ballot. For then incentives to free-ride will exist within the coalition. A single voter's optimal strategy is to urge the formation of a 200-vote coalition and then vote 100 himself. If all voters follow this strategy, we are left with honest preference revelation.<sup>6</sup>

But with small numbers of voters and publicly recorded votes, as in a representative body, the conditions for coalition formation are more favorable. This is particularly true because we usually elect representatives

<sup>3</sup> For further discussion of the problems raised by income effects or nonseparable utility functions, see Groves and Ledyard (1977b), Green and Laffont (1977a, 1979), and Laffont and Maskin (1980). For a defense of the assumption, see Tideman and Tullock (1977).

<sup>4</sup> For the most general discussion of this problem, see Hurwicz (1979).

<sup>5</sup> See Clarke (1971, 1977), Tideman and Tullock (1976), Tullock (1977a, 1982), Margolis (1982a), and Brubaker (1983).

<sup>6</sup> For further discussion, see Tideman and Tullock (1981).

as members of parties, which are natural coalition partners. Again we find ourselves confronted by a numerical dilemma: in a direct democracy with a large number of voters, no one has an incentive to gather information *or* join a coalition; in small committees of representatives, incentives exist to gather information about not only one's own preferences, but also those of others who may be potential coalition members.

**Bankruptcy:** Under the demand-revealing process it is possible for an outcome to emerge in which the entire private wealth of an individual is confiscated (Groves and Ledyard, 1977b, pp. 116–18). This is true of almost any voting procedure other than the unanimity rule, however, and is probably not a serious, practical problem. It does point out the need to view the process as taking place within some sort of system of constitutional guarantees and constraints upon the types of issues that come before the committee, however.<sup>7</sup>

Thus, the demand-revealing process is very much in the spirit of the Wicksellian approach to collective choice. Collective decision making is *within* a system of prescribed property rights, and *upon* a just distribution of income. The goal of collective action is to improve allocative efficiency, not to achieve distributive justice. Such redistribution as will take place is of the Pareto-optimal variety and is more appropriately viewed as part of the “allocation branch” of the public weal than of the “distribution branch.”<sup>8</sup>

### 8.1.2 *Vernon Smith's auction mechanism*

Vernon Smith (1977, 1979a,b) was the first to examine experimentally a simplified version of the demand-revelation process. In his experiments, each individual  $i$  announces both a bid,  $b_i$ , which is the share of the public good's cost that  $i$  is willing to cover, and a proposed quantity of the public good,  $G_i$ . The tax price actually charged  $i$  is the difference between the public good's costs,  $c$ , and the aggregate bids of the other  $n - 1$  voters,  $B_i$ ; that is,

$$t_i G = (c - B_i)G, \quad (8.7)$$

where  $B_i = \sum_{j \neq i} b_j$ , and  $G = \sum_{k=1}^n G_k/n$ . The procedure selects a quantity of public good only when each voter's bid matches his tax price and each voter's proposed public good quantity equals the mean:

$$b_i = t_i \text{ and } G_i = G, \text{ for all } i. \quad (8.8)$$

After each iteration of the procedure, voters are told what their tax prices and the public good quantity would have been had (8.8) been achieved at that iteration. If a voter's bid falls short of his tax price he can adjust either his bid or proposed public

<sup>7</sup> For further discussion of the bankruptcy issue, see Tullock (1977a), Tideman and Tullock (1977), and Groves and Ledyard (1977b,c).

<sup>8</sup> Tullock (1977d) has explored the redistributive potential of the process and claims somewhat more for it. On the distinction between Pareto-optimal redistribution and other kinds, see Hochman and Rodgers (1969, 1970).

good quantity to try to bring about an equilibrium. Only when all unanimously agree to both their tax prices and the public good quantity does the procedure stop.

At an equilibrium (8.8) is satisfied, and  $i$ 's utility can be written as

$$V_i = U_i(G) - T_i G, \quad (8.9)$$

where the utility from consuming  $G$  is expressed in money units. Maximizing (8.9) with respect to  $G_i$  we obtain the condition for  $i$ 's optimal proposed quantity for the public good,

$$\begin{aligned} dV_i/dG_i &= U'_i/n - t_i/n = 0 \\ U'_i &= t_i. \end{aligned} \quad (8.10)$$

Each voter equates his marginal utility from the public good to his tax price. Summing (8.10) over all voters, we obtain

$$\sum_{i=1}^n U'_i = \sum_{i=1}^n t_i = \sum_{i=1}^n (c - B_i) = c. \quad (8.11)$$

Equations (8.10) and (8.11) define the conditions for the Lindahl equilibrium.

The auction mechanism induces individuals to reveal their preferences for the public good by charging each voter a tax based not on his stated preference for the public good, but on the aggregate of all other stated preferences (bids). Each voter must be willing to make up the difference between the public good's costs at the aggregate bids of the other voters for the good to be provided. The ultimate incentive to state one's preferences honestly is provided by the knowledge that the good will not be provided unless all unanimously agree to a single quantity and set of tax prices.

Experiments by Smith (1977, 1979a,b, 1980) using this variant of the demand-revealing process indicated a fairly fast convergence on the Lindahl equilibrium. Harstad and Marrese (1982) also reported convergence to efficient outcomes in nine experiments with the Groves-Ledyard procedure. Thus, the vulnerability to individual strategizing of processes requiring sequential adjustment mechanisms may not be serious. The Public Broadcasting System has successfully employed another form of preference revelation procedure to allocate program space (Ferejohn, Forsythe, and Noll, 1979), and Tideman (1983) obtained some success with fraternity students using the demand-revealing process. These real-world experiments with demand-revealing procedures further buttress our confidence that its theoretical liabilities can be overcome in practice.

## 8.2 Point voting

We seek from a voting process two pieces of information: the quantity of the public good that satisfies the Pareto-optimality condition, and the set of tax shares that finances the purchase of this quantity. The demand-revealing process sidesteps the

second question by starting with a preassigned set of tax shares that suffice to cover the cost of supplying the public good. It induces honest preference revelation to determine the Pareto-optimal quantity of public good by means of the special incentive tax.

The need to charge a tax to induce honest preference revelation creates the problem of disposing of the revenue raised by the incentive tax under the one-step demand-revealing process, and makes the normative properties of the process dependent on the normative properties of the initial income distribution. These disadvantages can be avoided by giving each voter a stock of vote money that can be used to reveal preferences for public goods and has no other monetary value. No problem of disposing of the money collected exists, and the initial distribution of vote money can be made to satisfy any normative criterion one wishes. Hylland and Zeckhauser (1979) have proposed such a procedure.

The idea of giving citizens stocks of vote points and allowing them to allocate these points across the issue set in accordance with their preference intensities is not new.<sup>9</sup> The difficulty with point voting has always been that it does not provide the proper incentives for honest preference revelation, as Dodgson was well aware in the passage quoted at the beginning of this chapter. Individuals can better their realized outcomes by overstating their preferences on their most intense issues (Philpotts, 1972; Nitzan, Paroush, and Lampert, 1980; Nitzan, 1985). The important innovation of Hylland and Zeckhauser is their vote-point aggregation rule that provides voters with the proper incentive for honest preference revelation. They are able to show that with the appropriate determination of the vote points assigned to each citizen, voters reveal their true preferences for public goods when the government aggregates the *square roots* of the points of each voter. The main steps in this demonstration are outlined in the next section.

### **8.3\* An explication of the Hylland-Zeckhauser point-voting procedure**

We again assume the existence of preassigned tax shares for each citizen for each public good. Each citizen can calculate her total tax bill for each quantity of public good, and thus can determine the optimal quantities of each public good given her tax shares. This point-voting procedure, like the demand-revealing process, does not address the question of what the tax shares for each citizen should be. Its objective is to reveal preference intensities to determine the Pareto-optimal quantities of the public goods.

There are  $K$  public goods whose quantities must be determined. Each voter  $i$  is given a stock of vote points,  $A_i$ , to be allocated across the  $K$  public goods issues according to the voter's preference intensities. If voters wish to increase the quantity of the public good, they allocate a positive number of vote points to it; if they wish to decrease the quantity, they allocate a negative number of vote points. If  $|a_{ik}|$  is the absolute number of vote points that voter  $i$  allocates to issue  $k$ , then the  $a_{ik}$ s

<sup>9</sup> Dodgson's comment at the beginning of this chapter suggests that he did not invent the procedure, so it is probably over 100 years old. See more recent discussions by Musgrave (1959, pp. 130–1), Coleman (1970), Mueller (1971, 1973), Intriligator (1973), and Nitzan (1975).

must satisfy

$$\sum_{k=1}^K |a_{ik}| \leq A_i. \quad (8.12)$$

The government converts an individual's vote points into increments or decrements in the proposed quantity of public good using the rule

$$b_{ik} = f(a_{ik}), \quad (8.13)$$

where  $b_{ik}$  takes on the sign of  $a_{ik}$  and  $(b_{ik} = 0) \leftrightarrow (a_{ik} = 0)$ . The most straightforward rule is, of course,  $b_{ik} = a_{ik}$ , but, as we shall see, this rule does not provide the proper incentive for honest preference revelation. The quantities of public goods are determined through an iterative procedure. The government-auctioneer announces an initial proposal of public good quantities, perhaps the levels provided last year.

$$\begin{array}{c} G_1^0 \\ G_2^0 \\ \vdots \\ G_K^0. \end{array}$$

Each voter responds by stating an allocation of vote points across the  $K$  issues, which satisfies (8.12). If a voter wants a larger quantity of  $G_k$  than  $G_k^0$ , she allocates positive vote points to issue  $k$ , that is,  $a_{ik} > 0$ , and vice versa. The government determines a new vector of proposed public good quantities using (8.13); that is,

$$\begin{array}{l} G_1^1 = G_1^0 + \sum_{i=1}^n b_{i1} \\ G_2^1 = G_2^0 + \sum_{i=1}^n b_{i2} \\ \vdots \\ G_K^1 = G_K^0 + \sum_{i=1}^n b_{iK}. \end{array}$$

The process is repeated until a vector of public good quantities is obtained such that the aggregated votes for changing each public good quantity all sum to zero; that is,

$$\sum_{i=1}^n b_{ik} = 0, \quad k = 1, K. \quad (8.14)$$

There are three questions of interest concerning the procedure:

1. Does it converge?
2. What are the normative properties of the bundle of public goods quantities it selects?
3. What form does  $f()$  take?

Demonstrating that an iterative procedure converges is never an easy task. Hylland and Zeckhauser (1979) make a reasonable case for the convergence of this procedure, and we leave this issue aside.

The normative property we seek is Pareto optimality. This property is assured if we can choose a vector of public good quantities  $G = (G_1, G_2, \dots, G_K)$ , which maximizes

$$W(G) = \sum_{i=1}^n \lambda_i U_i(G), \quad (8.15)$$

where  $U_i(G)$  is voter  $i$ 's utility defined over the public good quantity vector  $G$  (see Chapter 2, Section 2.4\*). For  $W(G)$  to be at its maximum, the following first-order condition must be satisfied for each of the  $K$  public goods:

$$\sum_{i=1}^n \lambda_i \frac{\partial U_i}{\partial G_k} = 0, \quad k = 1, K. \quad (8.16)$$

The appropriately weighted marginal utilities must just balance, so that any change in  $G_k$  results in offsetting changes in weighted  $\partial U_i / \partial G_k$ s. We now have two conditions that our equilibrium vector of public goods must satisfy, (8.16) and (8.14). Clearly, we could ensure the Pareto optimality of any equilibrium vector to which the procedure converged, if

$$b_{ik} = \lambda_i \frac{\partial U_i}{\partial G_k}. \quad (8.17)$$

Then whenever convergence was achieved, that is,

$$\sum_{i=1}^n b_{ik} = 0, \quad k = 1, K,$$

(8.16) would also be satisfied, and Pareto optimality would be ensured. We now have a clue as to the form  $f()$  should take. It must be chosen to satisfy (8.17).

Now consider  $i$ 's decision for allocating her stock of vote points,  $A_i$ , at any step in the iterative procedure. She wishes to maximize her utility defined over the vector of public goods, given her vote-point budget constraint as given in (8.12); that is, she must at the  $t + 1$ th iteration maximize

$$O_i = U_i \left( G_1^t + \sum_{j \neq i} b_{j1} + b_{i1}, \dots, G_k^t + \sum_{j \neq i} b_{jk} + b_{ik} \dots \right. \\ \left. G_K^t + \sum_{j \neq i} b_{jK} + b_{iK} \right) + \mu_i \left( A_i - \sum_{k=1}^K |a_{ik}| \right). \quad (8.18)$$

The  $G_k^t$  are the announced quantities of public goods from the previous iteration and are fixed. The  $\sum_{j \neq i} b_{jk}$  are the aggregated vote points of the other voters on this iteration and are not subject to  $i$ 's control. Thus,  $i$  can change only the  $b_{ik}$ . Equation

(8.18) obtains a maximum when the following  $K$  equations are satisfied:

$$\frac{\partial U_i}{\partial G_k} f'(a_{ik}) = \mu_i, \quad k = 1, K \quad (8.19a)$$

when  $a_{ik} > 0$ , or

$$\frac{\partial U_i}{\partial G_k} f'(a_{ik}) = -\mu_i, \quad k = 1, K \quad (8.19b)$$

when  $a_{ik} < 0$ . Substituting for  $\partial U_i / \partial G_k$  in (8.17), we obtain

$$b_{ik} = f(a_{ik}) = \frac{\lambda_i \mu_i}{f'(a_{ik})} \quad (8.20)$$

when  $a_{ik} > 0$ . Now  $\lambda_i$  is the weight  $i$  gets in  $W$ , and  $\mu_i$  is the Lagrangian multiplier from (8.18). Thus,  $\lambda_i \mu_i = C$ , a constant. The function  $f(\cdot)$  must be such that

$$f(a_{ik}) f'(a_{ik}) = C. \quad (8.21)$$

From the observation that

$$\frac{df(a_{ik})^2}{da_{ik}} = 2f(a_{ik})f'(a_{ik}) \quad (8.22)$$

we obtain

$$\frac{df(a_{ik})^2}{da_{ik}} = 2C. \quad (8.23)$$

If we integrate (8.23), we obtain

$$f(a_{ik})^2 = 2Ca_{ik} + H, \quad (8.24)$$

where  $H$  is an arbitrary constant of integration. Setting  $H = 0$ , we obtain

$$f(a_{ik}) = \sqrt{2Ca_{ik}} = \sqrt{2\lambda_i \mu_i a_{ik}}. \quad (8.25)$$

Since  $\mu_i$  represents the marginal utility of a vote point to  $i$ ,  $\mu_i$  can be changed by changing  $i$ 's stock of vote points,  $A_i$ . In particular, if  $A_i$  is chosen such that

$$\mu_i = 1/(2\lambda_i), \quad (8.26)$$

then  $f(a_{ik})$  takes on the simple form

$$f(a_{ik}) = \sqrt{a_{ik}}. \quad (8.27)$$

The utility-maximizing vote-point allocations of each voter will be such as to maximize the weighted welfare function  $W$ , (8.15), for appropriately chosen  $A_i$ s, if the government-auctioneer determines the quantities of public goods by aggregating the square roots of each citizen's vote-point allocations. Taking the square root of vote-point allocations provides a sufficient penalty to overallocating vote points to more intense issues to offset the tendency to misrepresent preferences under naive point voting [ $f(a_{ik}) = a_{ik}$ ] mentioned earlier.

Note that an egalitarian assignment of vote points,  $A_i = A$  for all  $i$ , is consistent with giving each individual equal weight in the social welfare function,  $W$ , if and only if the marginal utility of a vote point is the same for all voters. This condition can, in turn, be interpreted as being equivalent to assuming that all voters have an equal stake, that is, an equal expected utility gain from collective action (Mueller, 1971, 1973; Mueller, Tollison, and Willett, 1975). Alternatively, an egalitarian assignment of vote points can be interpreted as an implicit decision to give lower weights ( $\lambda_i$ s) in the social welfare function to those with more intense preferences (higher  $\mu_i$ s).

The equilibrium obtained in the Hylland-Zeckhauser point-voting scheme is a Nash equilibrium, and strategizing on intermediary steps or coalitions could overturn the results. On the other hand, strategies for “beating the system” are not readily apparent.

#### **8.4 Voting by veto**

The demand-revealing and point-voting procedures call to mind analogies with market mechanisms in that real money or vote money is used to express preferences, and equilibrium is achieved through a tâtonnement process. The welfare properties of the procedures depend in part on the implicit interpersonal, cardinal utility comparisons that arise from aggregating dollar or point votes. In contrast, voting by veto (hereafter VV) utilizes only ordinal utility information.<sup>10</sup> Pareto optimality is achieved, as with the unanimity rule, through the rejection of Pareto-inferior outcomes. The procedure also resembles majority rule in important respects.

VV differs from the two procedures discussed earlier in this chapter in that it allows one to determine both the quantities of public goods and the tax shares to finance them. It differs from all voting procedures, as typically analyzed, in formally including the issue proposal process in the procedure, rather than assuming that voting takes place on a predetermined issue set.

The procedure has two steps. In the first, each member of the committee makes a proposal for the outcome of the committee process. These proposals could be the quantity of a single public good and the tax formula to finance it, or a whole vector of quantities of public goods with accompanying tax formulas. At the end of step 1, an  $n + 1$  proposal set exists consisting of the proposals of the  $n$  committee members and a status quo issue  $s$  (what was done last year, zero levels of all public goods, . . .). A random process is then used to determine an order of VV. The order of VV is announced to all members of the committee. The individual placed first in the veto sequence by the random process begins by eliminating (vetoing) one proposal from the  $n + 1$  element proposal set. The second veto-voter eliminates one proposal from the remaining  $n$  proposals. VV continues until all  $n$  members of the committee have vetoed one proposal. The one unvetoed proposal remaining in the issue set is declared the winner.

<sup>10</sup> This procedure was first discussed by Mueller (1978), with further development by Moulin (1979, 1981a,b, 1982) and Mueller (1984).



Table 8.3. *Rankings of issues in voting by veto example*

Issues	Voters		
	A	B	C
a	1	2	3(2)
b	3	1	2(3)
c	2	3	1
s	4	4	4

To see the properties of VV consider the following example for a committee of three. The voters,  $A$ ,  $B$ , and  $C$ , propose issues  $a$ ,  $b$ , and  $c$ , which together with  $s$  form the issue set. Let the individual preference orderings be as in Table 8.3, ignoring the two entries in parentheses.

Assume that each individual knows the other voters' preference orderings. Suppose that the randomly determined order of VV is  $A$ , then  $B$ , then  $C$ .  $A$  can make his proposal a winner by vetoing  $b$ . If  $B$  then vetoes either  $a$  or  $s$ ,  $C$  will veto the other issue in this pair ( $s$  or  $a$ ), and  $c$  wins. Because  $B$  prefers  $a$  to  $c$ ,  $B$ 's best strategy is to veto  $c$ , leaving  $C$  to veto  $s$ , making  $a$  the winner.

Now suppose that the randomly determined voting order is  $ACB$ .  $A$  no longer can get his proposal to win. If  $A$  vetoes  $c$ ,  $C$  vetoes  $a$  or  $s$ , and  $b$  wins. If  $A$  vetoes  $b$ ,  $C$  vetoes  $a$ , and  $c$  wins. Because  $A$  prefers  $c$  to  $b$ , he will veto  $b$ , leaving  $c$  to become the winner. The winners for the six possible permutations of voting sequences are as follows:

$$\begin{array}{ll} ABC \rightarrow a & BCA \rightarrow b \\ ACB \rightarrow c & CAB \rightarrow d \\ BAC \rightarrow a & CBA \rightarrow b. \end{array}$$

Each issue proposed by a committee member has a one-in-three chance of winning.

The preferences in Table 8.3 produce a cycle over  $a$ ,  $b$ , and  $c$  in pairwise voting under majority rule. Thus, in this opening example, the parallel between majority rule and VV seems close. Where the former produces a cycle over three issues, VV selects a winner at random with equal probability.

Now replace the two entries for  $C$  in Table 8.3 by those in parentheses; that is, assume that  $C$  now prefers  $a$  to  $b$ , all other rankings remaining the same. With this one change, the probability of  $a$ 's winning jumps to  $5/6$ . The only order of VV that selects a different issue than  $a$  is  $CAB$ , which leads to  $c$ 's victory.

This example illustrates an important incentive property of VV.  $A$  increases the probability of his proposal winning by advancing it in the preference ordering of another voter. Thus, the procedure establishes incentives to make proposals that, although perhaps favoring oneself, stand relatively high in the other voters' preferences. Of course, the same incentive exists for all voters, and a competition ensues to make the proposal standing relatively highest in all voters' preferences.

Table 8.4. *The elimination of proposals and voting by veto: example 2*

Voter	Rejects, $r_i$	Sets of possible winning proposals
$V_1$	$p_3$ or $p_2$ or $p_1$	$\{p_1\}$ or $\{p_2\}$
$V_2$	$p_4$ or $p_3$ or $p_2$	$\{p_1, p_2\}$ or $\{p_1, p_3\}$
$\vdots$		
$V_{n-3}$	$p_{n-1}$ or $p_{n-2}$ or $p_{n-3}$	$\{p_1, \dots, p_{n-4}, p_{n-3}\}$ or $\{p_1, \dots, p_{n-4}, p_{n-2}\}$
$V_{n-2}$	$p_n$ or $p_{n-1}$ or $p_{n-2}$	$\{p_1, \dots, p_{n-3}, p_{n-2}\}$ or $\{p_1, \dots, p_{n-3}, p_{n-1}\}$
$V_{n-1}$	$p_n$ or $p_{n-1}$	$\{p_1, \dots, p_{n-2}, p_{n-1}\}$ or $\{p_1, \dots, p_{n-2}, p_n\}$
$V_n$	$s$	$\{p_1, p_2, \dots, p_n\}$

The procedure can be shown to select a unique winning proposal out of any  $n + 1$  element proposal set, given the randomly determined VV sequence (Mueller, 1978, 1984). Moreover, the chances that an issue will win vary directly with its position in each voter’s ranking of the  $n + 1$  proposals. The lower a proposal is ranked by a voter, the lower are its chances of winning.

To see the latter point and further illustrate the properties of the procedure, consider the following example. A committee of  $n$  is offered a gift of  $G$  dollars if they can agree on a distribution of the gift. If they cannot agree, they retain the status quo distribution of nothing. Although the issue here is basically how to distribute  $G$ , the example resembles a public good decision under the unanimity rule in that all are better off only if they can all agree on a single proposal. The issue is one for which majority rule would produce a cycle. Let us examine the outcome under VV.

The initial, selfish instinct of a voter might be to propose that all of  $G$  go to himself and nothing to the other  $n - 1$  committee members. But this would make his proposal no better than the status quo and almost surely result in its defeat. He must offer some of  $G$  to the other voters.

What defeats a proposal is a low rank in another voter’s preference ordering. Thus, whatever amount of  $G$  a voter sets aside for the other committee members should be divided equally, since to discriminate against any one voter greatly increases the probability that this voter vetoes the proposal. Assuming that  $i$  selfishly desires a bit more of  $G$  for himself than he sets aside for others,  $i$ ’s proposal will look like the following:

$$\left( \frac{G}{n} - \frac{e_i}{n-1}, \frac{G}{n} - \frac{e_i}{n-1}, \dots, \frac{G}{n} + e_i, \dots, \frac{G}{n} - \frac{e_i}{n-1} \right). \quad (8.28)$$

Voter  $i$  proposes an egalitarian distribution of  $G$  with something extra for himself,  $G/n + e_i$ , and divides the remainder equally among the other  $n - 1$  voters, giving each  $G/n - e_i/(n - 1)$ . Assume that all proposals other than  $s$  take this form. We can now designate the proposals according to their degree of egalitarianism. Call  $p_1$  the proposal with the smallest  $e_i$  (that is, the most egalitarian),  $p_2$  the proposal with the second smallest  $e_i$ , and so forth. Assume no two proposals have the same  $e_i$ .

Now let the order of VV be determined as in Table 8.4.  $V_1$  is the first to vote,  $V_2$  the second, and so on. Once the VV sequence is determined, it is announced

to all voters. Given the nature of the proposals, any voter can easily determine the complete rankings of the  $n + 1$  proposals for all other voters. All voters rank the status quo proposal  $s$  last. All know that the last to go in the VV sequence,  $V_n$ , ranks  $s$  last. Given a choice between  $s$  and any other proposal,  $V_n$  rejects  $s$ . Thus, none of the voters will waste their veto on  $s$ , and  $s$  is left for  $V_n$  to veto. We can designate  $s$  with  $V_n$  as the proposal he definitely rejects. Considering  $V_n$  we can determine the set of possible winning issues as  $\{p_1, p_2, \dots, p_n\}$ . Voter  $V_{n-1}$  receives three proposals, one of which is  $s$ , and rejects the lower ranked of the two other proposals. Of the possible winning proposals,  $\{p_1, p_2, \dots, p_n\}$ ,  $V_{n-1}$  would veto the proposal ranked lowest by him in this set against any other proposal. Call this proposal  $r_{n-1}$ . If any voter who precedes  $V_{n-1}$  were to reject  $r_{n-1}$ , he would waste his veto. All will leave  $r_{n-1}$  for  $V_{n-1}$  to reject. Given the nature of the proposals, we can narrow the list of possible candidates for  $r_{n-1}$ .  $V_{n-1}$  ranks the least egalitarian of the proposals,  $p_n$ , lowest since it promises him the lowest payoff, unless  $p_n$  is his proposal. If  $V_{n-1}$  proposed  $p_n$ , he did not propose  $p_{n-1}$ , and ranks it lowest. Thus,  $V_{n-1}$  must reject either  $p_n$  or  $p_{n-1}$ .

Proceeding thus we can work our way up the list of voters, associating with each an issue to be rejected. If  $V_{n-1}$  proposed  $p_n$ , then  $V_{n-2}$  did not, and  $V_{n-2}$  rejects  $p_n$ . Considering both  $V_{n-1}$  and  $V_{n-2}$ , one or both did not propose  $p_n$ , and  $p_n$  is definitely rejected by one of the last three voters. Considering the last three voters,  $s$  and  $p_n$  are definitely eliminated as possible winning issues. As we work our way up the VV sequence, we discover that all proposals are eliminated as possible winners except  $p_1$  and  $p_2$ , the two most egalitarian proposals!

The most egalitarian proposal,  $p_1$ , wins most of the time because all voters other than its proposer rank it second to their own proposal. If the proposer of  $p_2$  happens to come first in the voting sequence (is  $V_1$ ), he can make his proposal the most egalitarian of the proposals by rejecting  $p_1$ ;  $p_2$  can win only if its proposer is  $V_1$ .<sup>11</sup> The probability that a given individual comes first in the voting sequence approaches zero as  $n$  increases, and thus the probability that any proposal other than the most egalitarian proposal wins approaches zero as the committee grows.

More generally, VV selects proposals ranked relatively high on all preference orderings. When the issue space is single-dimensional, and voter preferences single-peaked, VV assigns nonzero probabilities of winning to only the middle one-third proposals, with the highest probability going to the median proposal. This tendency to pick proposals "in the middle" is reinforced by the incentives facing voters at the proposal stage.

Let  $x$  and  $y$  be quantities of two public goods, or quality dimensions of a single public good to be decided by the committee. Let  $U_i(x, y)$  be  $i$ 's utility function reaching a maximum at some point  $I$  in the positive orthant. Assume circular indifference curves around  $I$ . Proposals take the form of combinations of  $x$  and  $y$ ,  $p_i(x_i, y_i)$ . The probability that any other voter  $j$  will reject  $p_i$  is higher the farther  $p_i$  is from  $j$ 's utility maximum,  $J_j$ ; call this probability  $\pi_j^i(x_i, y_i)$ . The probability

<sup>11</sup> Note that  $p_2$  does not always win when its proposer votes first. When he is followed by the proposer of  $p_3$ ,  $p_2$ 's proposer will not veto  $p_1$ , because then  $p_3$ 's proposer would veto  $p_2$ . Thus,  $p_1$  wins even when the proposer of  $p_2$  vetoes first, if this person is followed by the proposer of  $p_3$ .

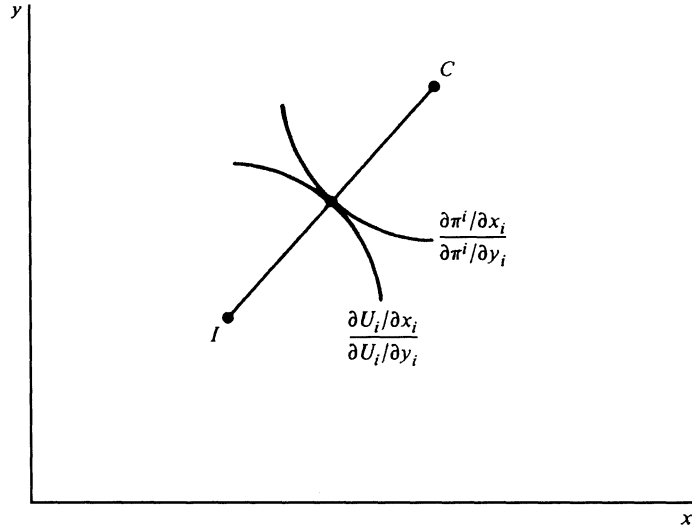


Figure 8.2. Determination of voter  $i$ 's proposal.

that any of the other  $n - 1$  voters will reject  $p_i$  is

$$\pi^i = \sum_{j \neq i} \pi_j^i. \tag{8.29}$$

Although  $\pi^i$  is not continuous, it is reasonable to assume that it approaches a continuous function with a minimum at  $C$ , the center of the distribution of peak utilities of the other  $n - 1$  voters, as  $n$  grows large. Let  $\bar{U}_i$  be  $i$ 's expected utility if his proposal does not win. His task is to propose a pair of characteristics  $(x_i, y_i)$  to maximize his expected utility,  $E(U_i)$ .

$$E(U_i) = (1 - \pi^i)U_i(x_i, y_i) + \pi^i \bar{U}_i. \tag{8.30}$$

Maximizing (8.30) with respect to  $x_i$  and  $y_i$  and setting each equation equal to zero, we derive

$$\begin{aligned} \frac{\partial U_i}{\partial x_i}(1 - \pi^i) - U_i \frac{\partial \pi^i}{\partial x_i} + \frac{\partial \pi^i}{\partial x_i} \bar{U}_i &= 0 \\ \frac{\partial U_i}{\partial y_i}(1 - \pi^i) - U_i \frac{\partial \pi^i}{\partial y_i} + \frac{\partial \pi^i}{\partial y_i} \bar{U}_i &= 0 \end{aligned} \tag{8.31}$$

from which we obtain

$$\frac{\partial U_i / \partial x_i}{\partial U_i / \partial y_i} = \frac{\partial \pi^i / \partial x_i}{\partial \pi^i / \partial y_i}. \tag{8.32}$$

Equation (8.32) defines a point of tangency between an indifference curve of  $i$  and an isoprobability of rejection locus around  $C$  (see Figure 8.2), a point on a pseudocontract curve running from  $i$ 's optimum point,  $I$ , to the center of the density function defined over the other voters' optima. In making a proposal,  $i$  is pulled

along this contract curve in the direction of  $C$  by the knowledge that the probability of his proposal's rejection is higher, the farther it lies from  $C$ . Application of VV will lop off the proposals lying farthest from the center of the density function defined over all optima, leaving as possible winners only a subset of proposals clustered around the center.

VV suffers from some of the same shortcomings as other procedures. As the number of participants grows, the incentive to participate declines. The process is also vulnerable to coalitions. If two of the three committee members in the preceding example could agree to discriminate against the third, they could combine redistributive elements into their proposals, making themselves better off, and the third person even worse off than under the status quo. The excluded member could veto but one of the proposals, and the other would win. As with other voting rules, however, the coalition problem is less important the larger the number of voters.

### 8.5 A comparison of the procedures

When Samuelson (1954, p. 182) proclaimed the task of revealing individual preferences for public goods impossible, he was assuming that a form of benefit tax would be used to finance the purchase of the public good. An individual's share of the costs of the public good would be tied to his stated preference for it. The demand-revealing process and point voting solve the preference-revelation problem by severing the link between stated preference and share of cost, as do other related processes like Smith's (1977) auction process.

Although these processes do not make a voter's share of the costs of a public good directly related to his stated preferences for it, they do impose a cost upon the voter for moving the committee outcome in a given direction. As Groves (1979, p. 227) observed, "The idea of a 'quid pro quo' is fundamental to an economic theory of exchange." With the exception of the logrolling models, the idea of a quid pro quo has not been part of either theoretical or real-world democratic processes; perhaps this explains their limited success at achieving Wicksell's goal of a *voluntary* exchange process of government. In most democratic procedures, votes are distributed as essentially free goods, with the only real constraint on their use being the ticking of the clock.

The procedures discussed in this chapter all break with this tradition in a fundamental way. The demand-revealing and point-voting schemes require that the voter be prepared to spend real money or fungible vote money to change the committee outcome. Under VV, vetoes are no longer free goods as they would be under the unanimity rule. Each individual has but one proposal to make, and one veto to cast.

Each of the procedures is also in the Wicksellian tradition in that the key equity issues are assumed to have been resolved prior to the application of the procedures.<sup>12</sup> For both the demand-revealing and point-voting procedures, the individual shares of the costs of the public good are predetermined. With demand revelation, the outcomes are further dependent on the initial distribution of income; with point

<sup>12</sup> For a discussion of this in the context of the demand-revealing process, see Tideman (1977).

voting, on the distribution of vote points. VV leaves aside the issue of initial income distribution.

Given a just starting point, the goal of collective action is to increase the welfare of all, and the task of the collective decision process is to indicate those situations where that is possible. The proposals differ, however, in the way that the gains from collective action are distributed. The demand-revealing process moves individuals out along their demand or offer curves to maximize the sum of consumer surpluses across individuals. The gains from collective action are distributed to those with the lowest shares of the public good costs and the highest initial incomes.<sup>13</sup> With point voting, the gains go to those with the lowest initial tax shares and highest initial stocks of vote points. With VV, the analogy is to the cake-cutting exercise, as brought about by the random determination of an order of VV. The gains from collective action will tend to be equal across individuals, and the normative characteristic of the process is set by this egalitarian property.

The Wicksellian voluntary exchange approach is ineluctably tied to philosophical individualism (Buchanan, 1949). Each individual enters the collective choice process to improve his own welfare, and the process is established so that all may benefit. Implicit here are a set of constitutional guarantees or constraints upon the collective decision process and, I believe, an assumption that coalitions of one group *against* another do not form. Each man strives *for* himself, but, as in the market, does not strive, collectively at least, *against* any other. The three proposals here all assume some form of constitutional constraints on the issues coming before the committee, and explicitly rule out coalitions. Under the demand-revealing process, the tax charged an individual is exactly equal to the cost that his participation in the process imposes on all others. Under VV, an individual can protect himself against a discriminatory threat to his well-being by any other voter's proposal through the veto he possesses.

In addition to the inherently individualistic orientation of these three proposals, they also resemble one another in the demands they place upon the individual who participates in the process. A simple yes or no will not do. The individual must evaluate in dollars his benefits under various possible alternatives, and, in the case of VV, also the benefits for other voters. This task is made easier by another Wicksellian characteristic of the procedures; each assumes that an expenditure issue and the tax to finance it are tied together. Although this latter feature might actually make the voter's decision task easier, the kind of information required of him under the three procedures is far more sophisticated than that obtained under present voting systems. It is also more sophisticated than one might expect "the average voter" to be capable of supplying, at least if one accepts the image of him gleaned from the typical survey data regarding his knowledge of candidates and issues. To many, the information required of voters will constitute a significant shortcoming of these processes. To me it does not. If we have learned one thing from the sea of work that has emerged following the classic contributions on public goods and democratic choice by Samuelson and Arrow, it is that the task of preference revelation in

<sup>13</sup> Tullock (1977b) has elaborated on the normative properties of the demand-revealing process.

collective decisions is not an easy one. If we must further assume that the individuals whose preferences we seek to reveal are only capable of yes or no responses, the task is hopeless from the start.

Much of the discussion of these procedures, pro and con, has been in the context of their use by the citizens themselves, as in a direct democracy. A more plausible application of them would appear to be by a committee of representatives, as in a parliament. Here the charge that the procedures are “too complicated” for the voters would carry less weight. Viewed as parliamentary procedures, both point voting and VV would appear to have an advantage over demand revelation, since they do not depend on the use of real money incentives. (Who pays the incentive tax, the citizens or the representatives?) The allocation of a representative’s vote points or the characteristics of his proposals under VV would also be useful information for voters when evaluating their representatives. Only the assumption that there are no coalitions would appear to constitute a problem, at least within a two-party system. With only two parties, for example, VV would yield the same outcomes as the simple majority rule. Both point voting and VV can be adapted for use in a multiparty parliamentary system, on the other hand, and both would have the advantage of allowing all parties to influence the outcomes rather than only those of the majority coalition, which forms “the government.”<sup>14</sup>

Although each has its weak points, these three procedures suggest that the knotty problem of preference revelation in collective choice can be resolved as both a theoretical and practical matter. Whether the optimal solution will be a variant on one of these processes or on a process yet to be discovered cannot at this point be ascertained. But the basic similarities running across these three processes are so strong, despite the inherently different procedural mechanics by which they operate, that one is led to suspect that these same characteristics will be a part of any “ultimate” solution to the preference revelation problem. And, if this is true, it further highlights Wicksell’s fundamental insight into the collective choice process.

#### *Bibliographical notes*

In addition to the procedures discussed in this chapter, mention should be made of those proposed by Thompson (1966), Drèze and de la Vallée Poussin (1971), and Bohm (1972).

<sup>14</sup> Mueller, 1996a, ch. 11.