CHAPTER 7

Simple alternatives to majority rule

My scheme is intended only for honest men.

Jean-Charles de Borda

Several alternatives to the majority rule have been proposed down through the years. Three of the newest and most complicated of these are presented in Chapter 8. Here we discuss some of the simpler proposals.

These voting procedures are usually not considered a means of revealing preferences on a public good issue, but a means of choosing a candidate for a given office. All issues cannot be chosen simultaneously. Only one of them can be. Although such choices are perhaps most easily envisaged in terms of a list of candidates for a vacant public office, the procedures might be thought of as being applied to a choice from among any set of mutually exclusive alternatives – such as points along the Pareto-possibility frontier.

7.1 The alternative voting procedures defined

Majority rule: Choose the candidate who is ranked first by more than half of the voters.

Majority rule, runoff election: If one of the m candidates receives a majority of first-place votes, this candidate is the winner. If not, a second election is held between the two candidates receiving the most first-place votes on the first ballot. The candidate receiving the most votes on the second ballot is the winner.

Plurality rule: Choose the candidate who is ranked first by the largest number of voters.

Condorcet criterion: Choose the candidate who defeats all others in pairwise elections using majority rule.

The Hare system: Each voter indicates the candidate he ranks *highest* of the m candidates. Remove from the list of candidates the one ranked highest by the fewest voters. Repeat the procedure for the remaining $m - 1$ candidates. Continue until only one candidate remains. Declare this candidate the winner.

The Coombs system: Each voter indicates the candidate he ranks *lowest* of the m candidates. Remove from the list of candidates the one ranked lowest by the most voters. Repeat the procedure for the remaining $m - 1$

candidates. Continue until only one candidate remains. Declare this candidate the winner.

Approval voting: Each voter votes for the k candidates ($1 \leq k \leq m$) he ranks highest of the m candidates, where k can vary from voter to voter. The candidate with the most votes is the winner.

The Borda count: Give each of the m candidates a score of 1 to m based on the candidate's ranking in a voter's preference ordering; that is, the candidate ranked first receives m points, the second one $m - 1, \ldots,$ the lowest-ranked candidate one point. The candidate with the highest number of points is declared the winner.

7.2 The procedures compared – Condorcet efficiency

This array of procedures is already lengthy and we could easily add to the list, although these cover the most frequently discussed procedures. Each has a certain intuitive appeal. How can one decide which is best?

There are several criteria for defining "best." First, we might define the axiomatic equivalents to each procedure, as we did with majority rule in Chapter 6, and compare the procedures on the basis of their axiomatic properties. These axioms are often rather abstract, however, and thus it may be somewhat difficult to declare procedure A superior to B just by looking at its axiomatic properties. We might declare one property most important, and compare the procedures on the basis of their ability to realize this property. The literature has proceeded in both ways, and we shall discuss the procedures in both ways.

The first of the axioms May (1952) requires of a voting procedure is that it is *decisive*; that is, it must pick a winner. Majority rule satisfies this criterion when there are but two candidates, a restriction May imposed on the problem. Choosing from a pair of alternatives is, however, the simplest *choice* one can conceptualize, and all of the above procedures select the same winner when $m = 2$. Interesting cases involve $m \geq 3$. With $m > 2$ no candidate may receive a majority of first-place votes, and no candidate may defeat all others in pairwise contests. Thus, when $m > 2$, both majority rule and the Condorcet criterion may declare no candidate a winner. Each of the other procedures will pick a winner.[1] Thus, for those who, on the basis of the arguments of Chapter 6, feel the majority rule ought to be the community's decision rule, interest in the other procedures arises only when $m > 2$.

Although the other procedures always pick a winner, even when a Condorcet winner does not exist, they do not always choose the Condorcet winner when one does exist. Table 7.1 presents a set of preference orderings for five voters in which X is the winner under the plurality rule, although Y is a Condorcet winner. Since a single vote for one's most preferred candidate is a possible strategy choice for voters under approval voting, X might also win under this procedure with the preference orderings of Table 7.1.

[1] We ignore ties. With large numbers of voters, ties are unlikely. The Borda count can easily be changed to accommodate ties in rankings (Black, 1958, pp. 61–4).

Table 7.1.

V_1	V_2	V_3	V_4	V_5
X	X	Y	Z	W
Y	Y	Z	Y	Y
Z	Z	W	W	Z
W	W	X	X	X

In Table 7.2, X is the Condorcet winner, while Y would be the winner by the Borda count. In Table 7.3, X is again the Condorcet winner, while issue W wins under the Hare system. Under each of the procedures other than majority rule, a winner may be chosen which is not the Condorcet winner even when the latter exists.

If one finds the properties of majority rule most attractive, then failure to select the Condorcet winner when one exists may be regarded as a serious deficiency of a procedure. One way to evaluate the different procedures is to compute the percentages of the time that a Condorcet winner exists and is selected by a given procedure. Merrill (1984, 1985) has made these percentage calculations and named them Condorcet efficiencies, that is, the efficiency of a procedure in actually selecting the Condorcet winner when one exists. Table 7.4 reports the results from simulations of an electorate of 25 voters with randomly allocated utility functions and various numbers of candidates.[2]

The first six rows report the Condorcet efficiencies for six of the procedures defined in Section 7.1. Voters are assumed to maximize expected utility under approval voting by voting for all candidates whose utilities exceed the mean of the candidates for that voter (Merrill, 1981). With two candidates, all procedures choose the Condorcet winner with efficiency of 100. The efficiency of all procedures is under 100 percent with three candidates. The biggest declines in efficiency in going from two to three candidates are for the plurality and approval voting procedures. When the number of candidates is as large as ten, the six procedures divide into three groups based on their Condorcet efficiency indexes: the Hare, Coombs, and Borda procedures all achieve about 80 percent efficiency; majority rule with one runoff and approval voting achieve about 60 percent efficiency; and the plurality rule selects the Condorcet winner only 42.6 percent of the time.

Table 7.2.

V_1	V_2	V_3	V_4	V_5
X	X	X	Y	Y
Y	Y	Y	Z	Z
Z	Z	Z	X	X

[2] Merrill (1984, p. 28, n. 4) reports that Condorcet efficiency is not very sensitive to the number of voters.

Table 7.3.

V_1	V_2	V_3	V_4	V_5
Y	W	X	Y	W
X	Z	Z	Z	X
Z	X	W	X	Z
W	Y	Y	W	Y

It is implausible to assume that an electorate would go to the polls nine separate times, as would be required under either the Hare or Coombs systems with 10 candidates. Therefore if either of these procedures were actually used, as a practical matter one would undoubtedly simply ask voters to write down their complete rankings of the candidates, and use a computer to determine a winner following the prescribed rule. Thus, the informational requirements of the Hare, Coombs, and Borda procedures are identical; they differ only in how they process this information. Given that they rely on the same information sets, it is perhaps not surprising that they perform about the same.

Of the six procedures listed in Table 7.4, the runoff and plurality procedures are the only ones in common use today. Thus, another way to look at the results of Table 7.4 is to calculate the gains in Condorcet efficiency in abandoning the plurality or runoff rule in favor of one of the other four procedures. The biggest gains obviously come in going to the Hare, Coombs, or Borda procedures, particularly if the number of candidates exceeds five. But much more information is demanded of the voter at the election. Approval voting might then be compared with the runoff and plurality system as a relatively simple procedure with Condorcet efficiency properties that exceed those of the plurality rule and approach those of the runoff system as the number of candidates expands. An important advantage of approval voting over the majority rule–runoff procedure is that approval voting requires that voters go to the polls only once (Fishburn and Brams, 1981a,b).

Table 7.4. *Condorcet efficiency for a random society (25 voters)*

	Number of candidates				
Voting system	3	4	5	7	10
Runoff	96.2	90.1	83.6	73.5	61.3
Plurality	79.1	69.4	62.1	52.0	42.6
Hare	96.2	92.7	89.1	84.8	77.9
Coombs	96.3	93.4	90.2	86.1	81.1
Approval	76.0	69.8	67.1	63.7	61.3
Borda	90.8	87.3	86.2	85.3	84.3
Social utility maximizer	84.6	80.2	77.9	77.2	77.8

Source: Merrill (1984, p. 28).

Table 7.5. *Utilitarian efficiency for a random society (25 voters)*

	Number of candidates				
Voting system	3	4	5	7	10
Runoff	89.5	83.8	80.5	75.6	67.6
Plurality	83.0	75.0	69.2	62.8	53.3
Hare	89.5	84.7	82.4	80.5	74.9
Coombs	89.7	86.7	85.1	83.1	82.4
Approval	95.4	91.1	89.1	87.8	87.0
Borda	94.8	94.1	94.4	95.4	95.9
Condorcet	93.1	91.9	92.0	93.1	94.3

Source: Merrill (1984, p. 39).

7.3 The procedures compared – utilitarian efficiency

Although the relative achievement of Condorcet efficiency may be an important property for those who favor majority rule as the voting procedure, for others it may not be the decisive factor in choosing a rule. Consider again Table 7.2. Issue X is the Condorcet winner. But this voting situation is clearly one that has some characteristics of a "tyranny of the majority." Under majority rule, the first three voters are able to impose their candidate on the other two, who rank him last. Y, on the other hand, is more of a "compromise" candidate, who ranks *relatively* high on all preference scales, and for this reason Y might be the "best" choice from among the three candidates. Y would be chosen under the Borda procedure, and under approval voting if any two of the voters (V_1, V_2, V_3) thought highly enough of Y to vote for both X and Y under approval voting, and not just for X. The closer Y stands to X, and the farther it stands from Z, the more likely it is that one of these voters will vote (X, Y) under approval voting and not just X.

An alternative normative criterion to that of Condorcet efficiency for a voting procedure is that it should maximize a utilitarian welfare function of, say, the form

$$W = \sum_i U_i, \qquad (7.1)$$

where the U_is are cardinal interpersonally comparable utility indexes for each voter i defined over the issue set. The bottom row of Table 7.4 reveals that the candidate whose choice would maximize (7.1) is the Condorcet winner only about 80 percent of the time. How, then, do the six procedures measure up against this utilitarian yardstick?

Table 7.5 presents further simulation results for a 25-person electorate. Note first that the Condorcet winner measures up rather well against the utilitarian maximum W criterion. But so, too, does the Borda count. It achieves a higher aggregate utility level for any number of candidates greater than two than the Condorcet winner would, if the Condorcet winner could always be found, or greater than any of the other five procedures would. Bordley (1983) presents analogous results. Although not providing full cardinal utility information, as is needed to achieve

100 percent efficiency in maximizing W, the Borda count, by providing a much richer informational base, is able to come fairly close to this objective.

Of additional interest in Table 7.5 is the performance of approval voting relative to the informationally more demanding Coombs and Hare systems. Given its performance by this utilitarian yardstick and its greater simplicity, we confine further attention to the Borda and approval voting procedures.

7.4 The Borda count

7.4.1 *Axiomatic properties*

Judged by the simulation results of Section 7.3, the Borda count would appear to be a potentially attractive voting procedure. What are its other normative properties?

Suppose we were to proceed as May (1952) did and seek an axiomatic representation of the Borda count. The first axiom May imposed was decisiveness – the procedure must be able to pick a winner from a binary pair. Some property like decisiveness is obviously attractive for any voting procedure. We can do this more formally by saying that we want the voting procedure to define a set of best elements, which we shall define as a choice set (Sen, 1970a, p. 10).

Definition of choice set: *An element x in S is a best element of S with respect to the binary relation R if and only if for every y in S, xRy. The set of best elements in S is called its choice set $C(S, R)$.*

Thus, we wish to have a voting rule that defines a choice set. Young (1974) proved that the Borda count was the only voting rule that defines a choice set and satisfies the four properties of neutrality, cancellation, faithfulness, and consistency.

As in May's theorem, the neutrality property is a form of impartiality with respect to issues or candidates. The names of the candidates or the nature of the issues do not matter.

The cancellation property, like anonymity in May's theorem, is a form of impartiality toward voters. Any voter i's statement "x is preferred to y" is balanced or canceled by any other voter j's statement "y is preferred to x"(Young, 1974, p. 45). What determines the social ordering of x and y is the number of voters who prefer x to y versus the number preferring y to x. The identities of the voters do not matter.

The faithfulness property is the totally innocuous condition that the voting procedure, when applied to a society consisting of only one individual, chooses as a best element that voter's most preferred element, that is, is faithful to that voter's preferences.

The above properties seem inherently reasonable. Indeed, they are all satisfied by majority rule. The more novel property is consistency.

Consistency: Let N_1 and N_2 be two groups of voters who are to select an alternative from the set S. Let C_1 and C_2 be the respective sets of alternatives

Table 7.6.

N_1			N_2			
V_1	V_2	V_3	V_4	V_5	V_6	V_7
z	x	y	z	z	x	x
x	y	z	x	x	y	z
y	z	x	y	y	z	y

that the two groups select using voting procedure B. Then if C_1 and C_2 have any elements in common (i.e., $C_1 \cap C_2$ is not empty), then the winning issue under procedure B when these two subgroups are brought together ($N_T = N_1 \cup N_2$) is contained in this common set of elements ($C_T = C_1 \cap C_2$).

This consistency property has obvious intuitive appeal. If two groups of voters agree on an alternative when choosing separately from a set of alternatives, they should agree on the same alternative when they are combined.

Majority rule also satisfies the consistency condition when the issue space and voter preferences are such as to ensure that a Condorcet winner always exists (Young, 1974, p. 44). Suppose, for example, that all issues were single dimensional and all voter preferences single peaked. Let m_1 be the median voter outcome for a committee of size N_1, where N_1 is odd. Let the interval $m_2 - m_2'$ be the choice set under majority rule for another committee of size N_2, where N_2 is even. If m_1 falls in the interval $m_2 - m_2'$, then m_1 will be the majority rule winner if the two committees combine, since one voter from N_1 has m_1 as a most preferred point, and $[(N_1 - 1)/2 + N_2/2]$ voters have preference peaks to the left of m_1 and the same number have peaks to the right of m_1. In this situation, majority rule satisfies the consistency property.

But we cannot always be sure that the conditions guaranteeing a Condorcet winner are satisfied. When they are not, then a cycle can arise of the form $xRyRzRx$. If in such situations we define the choice set as (x, y, z), the majority rule violates the consistency property, as the following example from Plott (1976, pp. 562–3) illustrates.

Let N_1 and N_2 be groups of voters with preference orderings as in Table 7.6. For N_1, a cycle over x, y, and z exists and we define the choice set as (x, y, z). For N_2, x and z tie and both beat y so its choice set is (x, z). The intersection of these two choice sets is (x, z) and the consistency criterion requires that x and z tie under majority rule when N_1 and N_2 are combined. But they will not tie. The committee $N_1 + N_2$ selects z as the unique winner using majority rule, thus violating the consistency condition.

An alternative way to look at the problem is to note that those versions of majority rule that do satisfy the consistency criterion, like the Condorcet principle, do not always define a nonempty choice set. Thus, if in going from two to three or more elements in our issue set, we wish the voting rule to continue to be capable of

Table 7.7.

V_1	V_2	V_3	V_4	V_5
X	X	X	Z	Z
Y	Y	Y	X	X
Z	Z	Z	W	W
W	W	W	Y	Y

picking a winner, and we wish to have the properties of neutrality, cancellation, faithfulness, and consistency, more information is required than is provided under the simple majority rule. Young's theorem demonstrates that the information needed is the complete preference ordering of every voter over the full issue set.[3]

7.4.2 *The Borda count and the "tyranny of the majority"*

In Section 7.3 we illustrated how the simple majority and plurality rules can lead to a "tyranny of the majority" in that a majority coalition gets its first choice over an alternative ranked relatively high by all voters. This sort of tyranny of the majority can be generalized.

Consider the set of voter preferences in Table 7.7. A coalition of the first three voters can impose its preferences on the community under the simple majority rule regardless of how the issues are presented to the voters. If the voters must choose from all four issues, the coalition imposes its first choice X. If the collective choice is restricted to the issues Y, Z, and W, the coalition imposes its first choice, Y, from among these three issues. Regardless of which combination of issues is presented to the voters, the coalition of the first three voters always gets its most preferred outcome.

X would also win under the Borda count if it were among the issues presented to the voters, but if for some reason X were an infeasible option and the voters had to choose among Y, Z, and W, Z would win under the Borda count. By taking into account more information about voter preferences, the Borda count can break a majority coalition's power to impose its will on the community over all possible sets of choices. Baharad and Nitzan (2001) prove that scoring rules like the Borda count, which take into account the preferences of voters over the full set of issues, are

[3] Nitzan and Rubinstein (1981) have replaced Young's faithfulness property with a monotonicity condition and proved an equivalence between these four axioms and the Borda count, where the Borda count now provides a complete ranking of all of the alternatives. The monotonicity condition can be stated as follows:

> **Monotonicity:** Let x and y be two distinct alternatives, and U and U' two sets of profiles of voter preferences. Suppose that the voting rule ranks x at least as good as y, xRy, under both sets of profiles U and U'. Let z be a third alternative such that for voter i, z is preferred to x (zP_ix) in U, but xP_iz in U'. Then the voting rule must designate x as strictly preferred to y (xPy) in U'.

This monotonicity condition demands that an alternative's relationship relative to a second alternative be strengthened if its status improves against some other third alternative.

superior to rules like the plurality and simple majority rules with respect to avoiding this sort of tyranny of the majority.[4]

7.4.3 *The Borda count and strategic manipulation*

Although the Borda count has axiomatic properties that seem at least the equal of majority rule, and it performs well when measured by the yardsticks of the utilitarian welfare function or of avoiding tyrannous majorities, its Achilles' heel is commonly felt to be its vulnerability to strategic behavior (Pattanaik, 1974; M. Sen, 1984). Consider again Table 7.2. Issue Y wins using the Borda count when all voters vote sincerely. If the first three voters were to state their rankings of the issues as $X P_i Z P_i Y$, however, the Borda count would select X as the winning issue. Thus, an incentive exists for voters 1 to 3 to misstate their preferences, *if they know the preferences of other voters and expect the other voters to vote sincerely.*

With three or more issues *all* voting procedures can be manipulated by one voter's misstating her preferences, however, so the relevant question to ask of a voting procedure is whether it is *more* susceptible to manipulation than other procedures.[5] Saari (1990) has attempted to answer this question by examining all possible preference orderings with committees of three or more members, and three or more issues. Saari constructs a measure of *micro*manipulability, which is the percentage of the situations in which one person or a small coalition could make themselves better off by misstating their preferences under a given voting rule. He finds that among the most popular choices of voting rules, like those examined in this chapter, the Borda count performs the best, either minimizing or coming close to minimizing the likelihood of successful manipulation.

If one group of voters can behave strategically, so can another. If voters 4 and 5 in Table 7.2 suspect that the other voters are trying to manipulate X's victory, they can try to avoid having their worst alternative, X, win by misstating their preferences as $Z P Y P X$. With both groups of voters now misstating their preferences, Z wins under the Borda rule. Thus, voters 1 to 3 take a chance when they raise Z above Y in their stated preference orderings of bringing about not X's victory, but Z's. The Borda count satisfies a nonnegativity or monotonicity condition (J.H. Smith, 1973). Lifting Y above Z in a voter's stated preference, ordering either raises or leaves unchanged Y's position in the social ordering, while having the reverse effect on Z. A risk-averse voter, uncertain of the relative chances of X, Y, and Z winning, either due to ignorance of other voter preferences or uncertainty about their possible strategic behavior, maximizes her expected utility under the Borda procedure by honestly stating her true ranking of the three issues.

As the electorate grows large the likelihood of a voter's knowing the preferences of the others grows small, and thus so do the chances of successfully manipulating

[4] The properties of another scoring rule – point voting – are addressed in the next chapter.

[5] The main theorems about the potential for strategic manipulation of all voting procedures were first proved by Gibbard (1973) and Satterthwaite (1975). Their results are discussed in Chapter 24.

the outcome. Moreover, the probability of any one voter's vote being decisive also declines, of course. Thus, the likelihood of successful strategic manipulation of the outcomes under the Borda count will decline as the number of voters increases.[6]

7.5 Approval voting

With large numbers of alternatives, the Borda procedure has the possible disadvantage of complexity. The voter must list her complete ranking of the set of alternatives, which with fairly large issue sets could discourage individuals from voting.

In contrast, approval voting asks voters only to draw a line through their preference ordering so as to separate the candidates into those they approve of and those they do not. If the candidates are relatively evenly spaced from one another in terms of expected utility payoffs, then this line will divide the set of candidates roughly into two equal-sized groups (Merrill, 1981). Voters need not concern themselves with how the two sets of candidates stack up against one another within the approval and disapproval sets.

When the number of candidates is few, or voters are indifferent between various pairs of candidates, approval voting also has some advantages over other procedures in discouraging strategic behavior. Brams and Fishburn (1978) have proven that when voter preferences are dichotomous in the sense that it is possible for every voter i to divide the set of all candidates S into two subsets, S_{i1} and S_{i2}, such that i is indifferent among all candidates in S_{i1}, and among all in S_{i2}, then under approval voting there is a single undominated strategy – vote for all candidates in the subset S_{ij} who are ranked higher than those in the other subset. Approval voting is the only voting procedure to have a unique, undominated strategy for all possible dichotomous preference relationships.

When voter preferences are trichotomous – that is, candidates are divided into three indifference groups, S_{i1}, S_{i2}, S_{i3} – then the only undominated strategies under approval voting are to vote sincerely for either (1) all candidates in the most preferred group or (2) all candidates in the two most preferred groups. Approval voting is the only voting system that is sincere in this sense for every possible trichotomous preference relationship.

When voter preferences are multichotomous – that is, four or more indifference groups are required – no voting procedure is sincere or strategy-proof for all possible multichotomous preference relationships.

Since all procedures discussed in this chapter are identical to majority rule when there are only two candidates, the importance of the results for dichotomous candidates rests on the plausibility of assuming voter indifference between various pairs of candidates in a multicandidate race. On this issue opinions differ (Niemi, 1984). Approval voting proved to be more susceptible to micromanipulation than the Borda count in Saari's (1990) comparisons.

[6] Holding the number of alternatives fixed. Conversely, the potential for manipulation rises as the number of alternatives increases (Nitzan, 1985).

Table 7.8. *Delegate totals under various decision rules*

Candidate	Plurality rule	Double election	Condorcet choice	Borda count	Adjusted[a] Borda count
McGovern	1,307	766	766	766	584
Muskie	271	788	869	869	869

[a] Adjusted Borda count is modified to allow for ties. See Black (1958, pp. 61–4).
Source: Joslyn (1976, Table 5, p. 12).

Beyond whatever advantages it possesses in discouraging strategic behavior, however, approval voting deserves serious attention as a possible substitute for the plurality and majority rule–runoff rules because of its superior performance, as judged by the Condorcet or utilitarian efficiency criteria, and greater simplicity than the Hare, Coombs, Borda, and to some extent majority rule–runoff procedures.

7.6 Implications for electoral reform

State presidential nominating elections and elections of representatives to the House and Senate in the United States are based on a first-past-the-post criterion, that is, the plurality rule. Yet the plurality rule scores worst by the Condorcet and utilitarian efficiency criteria. This observation has led to recommendations that an alternative rule be introduced, particularly in presidential primaries where the number of candidates may be large (Kellett and Mott, 1977).

The possible significance of such a reform is revealed in Joslyn's (1976) study of the 1972 Democratic presidential primaries. Joslyn argued that the plurality rule favored extremist candidate George McGovern, who was the first choice of a plurality of voters in many states but was ranked relatively low by many other voters, over "middle-of-the-road" Edmund Muskie, who was ranked relatively high by a large number of voters. Joslyn's most striking result is his recalculation of final delegate counts under the various voting rules presented in Table 7.8 (double election is a two-step runoff procedure). The interesting feature of this table is the dramatic increase in Muskie's delegate strength under *any* of the voting procedures other than the plurality rule.[7]

One might argue that Muskie *should* have been the Democratic party's nominee in 1972 and that, therefore, one of the other voting procedures is preferable to the plurality rule. Muskie would have had a better chance to defeat Nixon than McGovern, and McGovern's supporters would probably have preferred a Muskie victory to a McGovern defeat in the final runoff against Nixon. And, with the infinite wisdom of hindsight, one can argue that "the country" would have been better off with a Muskie victory over Nixon.

The rules of the game do matter.

[7] Muskie would undoubtedly also have faired much better against McGovern had approval voting been used. See Kellett and Mott (1977) and Brams and Fishburn (1978, pp. 840–2).

Bibliographical notes

The seminal discussion of the various voting rules is by Black (1958, pp. 55–75). Black also presents biographical discussions of the work of the Marquis de Condorcet (pp. 159–80) and Jean-Charles de Borda (pp. 156–9, 178–90). See also Young's 1988 article and his 1997 survey.

The Borda count is also discussed by Plott (1976, pp. 560–3), Sen (1982, pp. 187–7, 239–40, 376–7), and Schwartz (1986, pp. 179–81). Saari (1994) develops a new geometric methodology to examine the properties of voting rules. In addition to reestablishing many of the known properties of the various voting rules, like cycling under the majority rule, Saari uncovers several attractive features of the Borda count with his new methodology.

The properties of approval voting were first discussed by Brams (1975, ch. 3) with important extensions presented by Brams and Fishburn (1978) and Fishburn and Brams (1981a,b). The major results on approval voting are pulled together in their book (Brams and Fishburn, 1983).