CHAPTER 5

Majority rule – positive properties

> But as unanimity is impossible, and common consent means the vote of the majority, it is self-evident that the few are at the mercy of the many.
>
> John Adams

5.1 Majority rule and redistribution

As Chapter 4 indicated, a committee concerned only with providing public goods and correcting for externalities might nevertheless choose as its voting rule the simple majority rule, if it placed enough weight on saving time. But speed is not the only property that majority rule possesses. Indeed, once issues can pass with less than unanimous agreement, the distinction between allocative efficiency and redistribution becomes blurred. Some individuals are inevitably worse off under the chosen outcome than they would be were some other outcome selected, and there is in effect a redistribution from those who are worse off because the issue has passed to those who are better off.

To see this point more clearly, consider Figure 5.1. The ordinal utilities of two groups of voters, the rich and the poor, are depicted on the vertical and horizontal axes. All of the members of both groups are assumed to have identical preference functions. In the absence of the provision of any public good, representative individuals from each group experience utility levels represented by S and T. The point of initial endowment on the Pareto-possibility frontier with only private good production is E. The provision of the public good can by assumption improve the utilities of both individuals. Its provision thus expands the Pareto-possibility frontier out to the curve $XYZW$. The segment YZ corresponds to the contract curve in Figure 4.3, CC'. Under the unanimity rule, both groups of individuals must be better off with the provision of the public good for them to vote for it. So the outcome under the unanimity rule must be a quantity of public good and tax share combination, leaving both groups somewhere in the YZ segment along the Pareto-possibility frontier.

But there is no reason to expect the outcome to fall in this range under majority rule. A coalition of the committee's members can benefit by redefining the issue to increase their benefits at the expense of the noncoalition members, say, by shifting the tax shares to favor the coalition. If the rich were in the majority, they could be expected to couple the public good proposal with a sufficiently regressive tax package so that the outcome wound up in the XY segment. If the poor were in the majority, the taxes would be sufficiently progressive to produce an outcome in ZW.

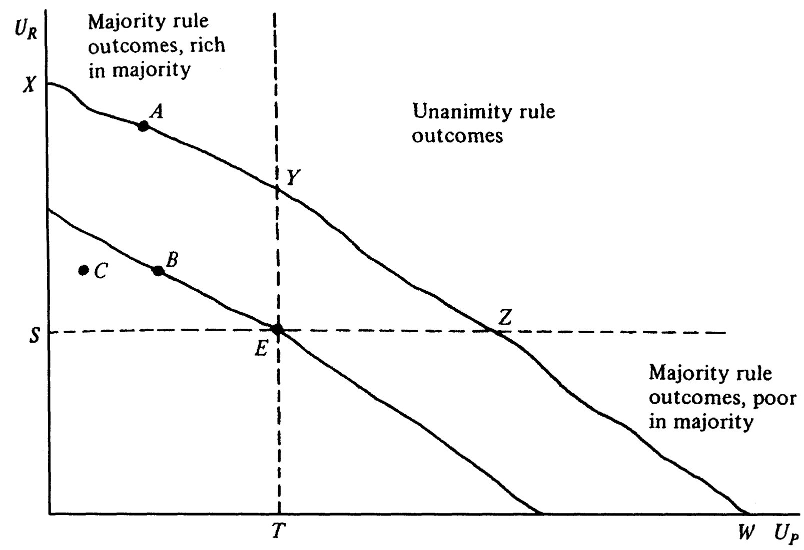

Figure 5.1. Outcomes under the unanimity and the simple majority rule.

Given the opportunity to redefine the issue proposed through the alteration of either the quantity of the public good provided, the tax shares, or both, one can expect with certainty that the outcome of the collective choice process *will* fall outside of the Pareto-preferred segment *YZ* (Davis, 1970). As long as the issue could be continually redefined in such a way that a majority still benefited, it would pass, and a stable majority coalition could, in principle, push a minority back as far along the Pareto-possibility frontier as their consciences or the constitution allowed.

The process of transforming a proposal unanimously supported into one supported by only a simple majority resembles that described by Riker (1962), in which "grand" coalitions are transformed into minimum winning coalitions. In developing his theory of coalitions, Riker makes two key assumptions: (1) decisions are made by majority rule and (2) politics is a zero-sum game. He assumes that the allocational efficiency decisions (quantities of public goods) are all optimally resolved as a matter of course, and that the political process is left with the distributional issue of choosing from among the Pareto-efficient set (pp. 58–61). Thus, Riker (1962, pp. 29–31) takes the extreme position that politics involves *only* redistribution questions, and is a pure zero-sum game. Given that the game is to take from the losers, the winners can obviously be better off by increasing the size of the losing side, as long as it remains the losing side. Under majority rule, this implies that the losing coalition will be increased until it is almost as large as the winning coalition, until the proposal passes by a "bare" majority. In Riker's description, the committee is made up of several factions or parties of different sizes, rather than two "natural" coalitions, as depicted earlier, and the process of forming a minimum winning coalition consists of adding and deleting parties or factions until two "grand" coalitions of almost equal size are formed. In regular committee voting, the process would consist of adding and deleting riders to each proposal, increasing the number of losers, and increasing the benefits to the remaining winners.

Several writers have described ways in which majority rule can lead to redistribution other than via the obvious route of direct cash transfers. The pioneering effort in this area was by Tullock (1959). Tullock described a community of 100 farmers in which access to the main highway is via small trunk roads, each of which serves only 4 or 5 farmers. The issue comes up as to whether the entire community of 100 should finance the repair of all of the trunk roads out of a tax on the entire community. Obviously one can envisage a level of repairs and set of taxes on the individual farmers under which such a proposal would be unanimously adopted. But under majority rule it is to the greater advantage of some to propose that only one half of the roads are repaired out of a tax falling on the entire population. Thus, one can envisage a coalition of 51 of the farmers forming and proposing that only the roads serving them are repaired out of the community's general tax revenue (Tullock discusses other possible outcomes, which we take up shortly). Such a proposal would pass under majority rule, and obviously involves a redistribution from the 49 farmers who pay taxes and receive no road repairs to the 51 farmers whose taxes cover only slightly more than one half of the cost of the road repair.

In the Tullock example, redistribution to the 51 farmers in the majority coalition takes place through the inclusion in the entire community's budget of a good that benefits only a subset of the community. Each access road benefits only 4 or 5 farmers and is a public good with respect to only theses farmers. The optimal size of jurisdiction for deciding each of these "local" public goods would seem to be the 4 or 5 farmers on each access road. The inclusion of private goods in the public budget as a means of bringing about redistribution was first discussed by Buchanan (1970, 1971) and has been analyzed by several other writers. Building on Buchanan's papers, Spann demonstrated that the collective provision of a private good financed via a set of Lindahl tax prices leads to a redistribution from the rich to the poor (Spann, 1974). To see this, consider Figure 5.2. Let D_P be the demand schedule for the poor and D_R for the rich. Let X be a pure private good with price = marginal social cost = P_X. If the good is supplied to the market privately, the poor purchase X_P at price P_X; the rich purchase X_R. Assume next that the good is collectively purchased and supplied to the community in equal quantities per person, as if it were a public good. The optimal quantity of X is then given by the intersection of the community demand schedule, obtained by vertically summing the individual demand schedules. (We ignore here income effect considerations. The argument is not substantively affected by this omission.)

The supply schedule under collective provision is obtained by multiplying the market price of the good by the number of members of the community. If we assume for simplicity an equal number of rich and poor, the community will purchase X_C units of the good for each individual. At this quantity, a poor individual places a marginal evaluation on the good of X_CH, and his Lindahl tax is t_P. A member of the rich group pays t_R. In effect, the poor receive a subsidy of $ACHt_P$, the difference between the price they pay for the good and its social cost multiplied by the quantity they consume. But their consumer surplus gain from the collective provision of the private good is only $ABHt_P$. Thus, there is a deadweight loss of BCH through the collective provision of X. In addition to the direct transfer of income from R

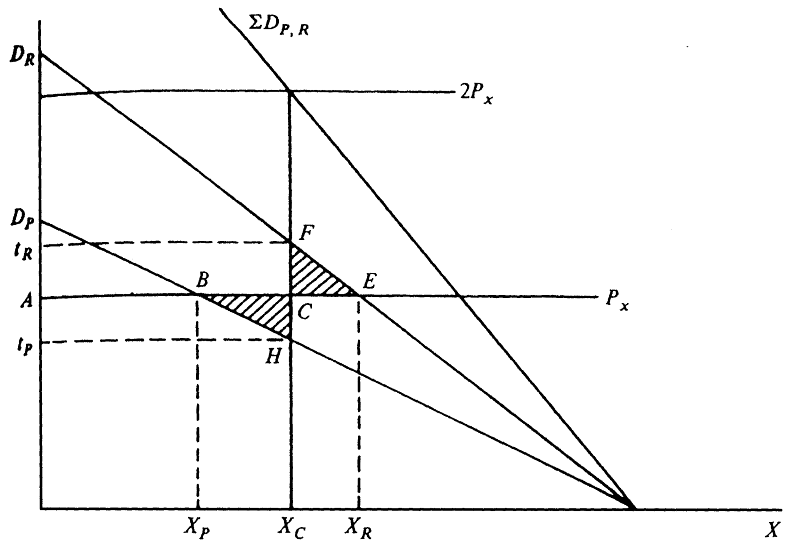

Figure 5.2. Redistribution with the public provision of a private good.

to P (t_PHCA) via the subsidization of P's purchase of X, R is worse off by being forced to consume a less than optimal amount of X. R loses the consumer surplus triangle FCE.

This loss in efficiency comes about through the constraint placed on each individual's behavior, when all are forced to consume the same quantity of the private good. Given the costs of producing the private good, all could be made better off by being allowed to maximize their individual utilities at the set of market prices for this and the other goods. The additional constraint that all consume the same quantity lowers the set of attainable utilities. But the poor are better off receiving the redistribution in this form than not at all, and if it is not feasible for them to obtain direct cash subsidies via lump-sum transfers, and it is possible to obtain them through the collectivization of private good supply, then the latter is worth pursuing.

The inefficiency brought about by constraining the rich to consume less than their most preferred quantity of X can be removed by allowing them to purchase additional units in the market. Most governments that publicly provide housing, medical care, education, and similar goods that could be provided by the market do allow individuals to supplement what they receive from the state, or to opt out of the system entirely. When upper income groups pay to send their children to private schools as they do in the United States and the United Kingdom, or obtain health care from private physicians rather than from the free National Health Service in the United Kingdom, an additional form of redistribution from rich to poor occurs as the well-to-do pay part of the costs of the publicly provided good, but consume none of it. Although allowing the rich to purchase the private good on the market reduces the efficiency loss from providing this private good publicly, it does not eliminate it entirely, as those remaining in the program continue to be forced to purchase the private good under the artificially imposed constraint of an equal quantity and/or uniform quality (Besley and Coate, 1991).

The inefficiency also remains when the upper income groups continue to use the publicly provided service, but supplement their purchases on the market. If the quantity (quality) of the public service is chosen using the simple majority rule, the chosen quantity or quality may be greater than that which both the rich and the poor prefer. The poor oppose the collective choice because they are forced to consume more of the publicly provided good than they wish, given their tax price; and the rich too would prefer to pay less in taxes, consume less of the publicly provided service, and purchase more in the market.[1]

Where publicly provided education at the elementary school level redistributes income from the highest to the lowest income groups, publicly provided higher education redistributes from the lowest to the middle income groups, and where professional education in law, medicine, and business is freely provided by the state – as it is throughout most of Europe – redistribution is from the average taxpayer to those who will soon join the highest income groups in society.[2]

As the pattern of governmental transfers depicted in Table 3.5 of Chapter 3 reveals, all redistribution is not from rich to poor, nor even predicated on differences in incomes. Occupation, sex, race, geographic location, recreational preferences, and political affiliation can all be used to delineate the targets of redistribution. What is required for redistribution to take place under majority rule is that the members of the winning coalition be clearly identifiable, so that the winning proposal can discriminate in their favor, either on the basis of the distribution of the benefits it provides (for example, Tullock's unequal distribution of roads at equal taxes) or the taxes it charges (for example, Buchanan and Spann's equal quantities of private good X at unequal taxes).

Regardless of what form it takes, and regardless of whether political choice under majority rule is a pure zero-sum game, as Riker assumes, or involves allocational efficiency changes *plus* redistribution, the fact remains that the redistributional characteristics of any proposal will figure in its passage, and that majority rule creates the incentive to form coalitions and redefine issues to achieve these redistributional gains. Indeed, from the mere knowledge that an issue passed with some individuals in favor and others opposed, one cannot discern whether it really was a public good shifting the Pareto-possibility frontier out to $XYZW$ in Figure 5.1 coupled to a tax unfavorable to the poor, say, resulting in an outcome at A; a pure redistribution along the private-good Pareto-efficiency frontier resulting in B; or an inefficient redistribution from the poor to the rich via the collective provision of a private good resulting in, say, C. All one can say with much confidence is that the rich appear to believe that they will be better off, and the poor that they will be worse off from passage of the proposal; that is, the move is into the region $SEYX$.

Thus, even if the emergence of states is better explained as cooperative efforts undertaken to benefit all members of the community rather than as a power move by one group in society to exploit the rest, it is now clear that the use of the majority rule to make collective decisions must transform the state at least in part into a

[1] Gouveia (1997). This result relies on the median voter theorem introduced in Section 5.3* of this chapter.

[2] The allocational (in)efficiency and redistribution properties of education are discussed by Barzel (1973) and Barzel and Deacon (1975).

redistributive state. Since all modern democracies use the majority rule to a considerable degree to make collective decisions – indeed the use of the majority rule is often regarded as the mark of a democratic form of government – all modern democratic states must be redistributive states in part, if not in toto.

5.2 Cycling

Given that majority rule must induce some element of redistribution into the collective decision process, we take up next an attribute of majority rule when a pure redistribution decision is to be made. Consider a three-person committee that must decide how to divide a gift of $100 among them using majority rule. This is a pure distributional issue, a simple zero-sum game. Suppose that V_2 and V_3 first vote to divide the $100 between themselves, 60/40. V_1 now has much to gain from forming a winning coalition. He might propose to V_3 that they split the $100, 50/50. This is more attractive to V_3, and we can expect this coalition to form. But now V_2 has much to gain from trying to form a winning coalition. He might now offer V_1 a 55/45 split forming a new coalition, and so on. When the issues proposed involve redistribution of income and wealth, members of a losing coalition always have a large incentive to attempt to become members of the winning coalition, even at the cost of a less-than-equal share.

The outcome of a 50/50 split of the $100 between a pair of voters is a von Neumann-Morgenstern solution to this particular game (Luce and Raiffa, 1957, pp. 199–209). This game has three such solutions, however, and there is no way to predict which of these three, if any, would occur. Thus, the potential for cycles, when issues involve redistribution, seems quite large. It is always possible to redefine an issue to benefit one or more members and harm some others. New winning coalitions containing some members of the previously losing coalition and excluding members of the previously winning coalition are always feasible. But, as we have seen from the discussion of majority rule, when issues can be amended in the committee, any pure allocative efficiency decision can be converted into a combination of a redistribution and an allocative efficiency change via amendment. Thus it would seem that when committees are free to amend the issues proposed, cycles must be an ever-present danger.

The possibility that majority rule can lead to cycles across issues was recognized over two hundred years ago by the Marquis de Condorcet (1785). Dodgson (1876) analyzed the problem anew one hundred years later, and it has been a major concern of the modern public choice literature beginning with Black (1948b) and Arrow (1951, rev. ed. 1963).[3] Consider the following three voters with preferences over three issues, as in Table 5.1 ($>$ implies preferred). X can defeat Y, Y can defeat Z, and Z can defeat X. Pairwise voting can lead to an endless cycle. The majority rule can select no winner nonarbitrarily.[4]

If we define Z as a payoff to voters V_2 and V_3 of 60/40, Y as the payoff (50, 0, 50), and X as (55, 45, 0), the ordinal rankings of issues in Figure 5.3 correspond

[3] For a discussion of these and other early contributions, see Black (1958), Riker (1961), and Young (1997).

[4] See A.K. Sen's discussion (1970a, pp. 68–77).

Table 5.1. *Voter preferences that induce a cycle*

	Issues						
Voters	X		Y		Z		X
1		>		>		<	
2		>		<		>	
3		<		>		>	
Community		>		>		>	

to the zero-sum pure distribution game. But it is also possible to get orderings as in Table 5.1 and Figure 5.3 for issues involving allocational efficiency. If X, Y, and Z are sequentially higher expenditures on a public good, then the preferences of Voters 1 and 3 can be said to be single-peaked in the public good–utility space (see Figure 5.3). Voter 2's preferences are double-peaked, however, and herein are a cause of the cycle. Change 2's preferences so that they are single-peaked, and the cycle disappears.

One of the early important theorems in public choice was Black's (1948a) proof that majority rule produces an equilibrium outcome when voter preferences are single-peaked. If voter preferences can be depicted along a single dimension, as with an expenditure issue, this equilibrium lies at the peak preference for the median voter. Figure 5.4 depicts the single-peaked preferences for five voters. Voters 3, 4, and 5 favor m over any proposal to supply less. Voters 3, 2, and 1 favor it over proposals to supply more. The preference of the median voter decides.

5.3* The median voter theorem – one-dimensional issues

The proof follows Enelow and Hinich (1984, ch. 2). The two key assumptions for the median voter theorem are (1) that issues are defined along a single dimensional

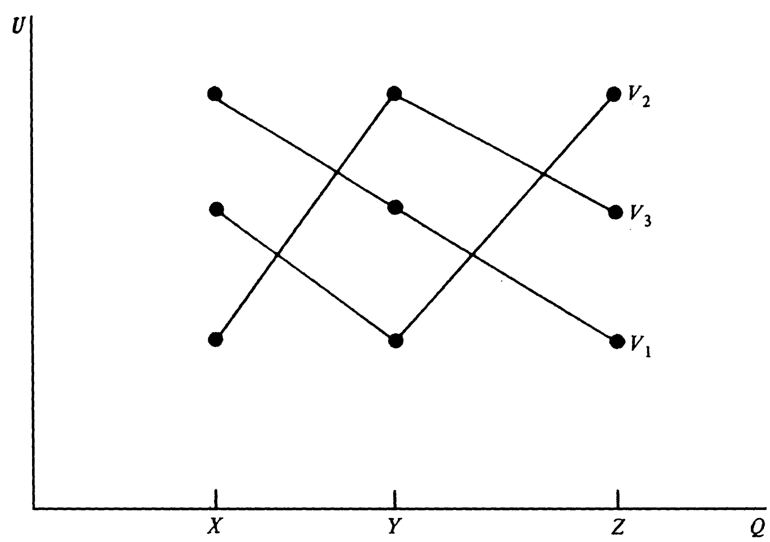

Figure 5.3. Voter preferences that induce a cycle.

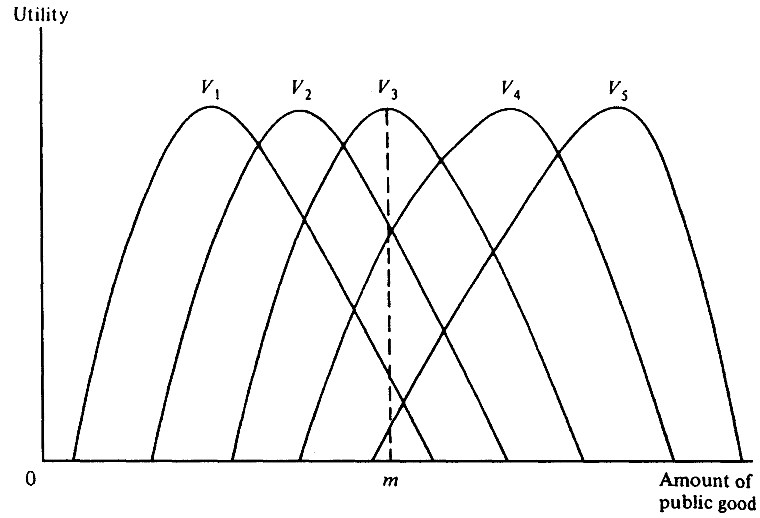

Figure 5.4. The median voter decides.

vector x and (2) that each voter's preferences are single-peaked in that one dimension. Let voter i's preferences be represented by a utility function $U_i()$ defined over x, $U_i(x)$. Let x_i^* be voter i's most preferred point along the x vector. Call x_i^* i's ideal point.

Definition: *x_i^* is i's ideal point if and only if (iff) $U_i(x_i^*) > U_i(x)$ for all $x \neq x_i^*$.*

Definition: *Let y and z be two points along the x dimension, such that either $y, z \geq x_i^*$ or $y, z \leq x_i^*$. Then voter i's preferences are single-peaked iff $[U_i(y) > U_i(z)] \leftrightarrow [|y - x_i^*| < |z - x_i^*|]$.*

In other words, the definition of single-peaked preferences says that if y and z are two points on the same side of x_i^*, then i prefers y to z if and only if y is closer to x_i^* than z is. If all preferences are single-peaked, then preferences like those of Voter 2 in Figure 5.3 cannot occur (note z is 2's ideal point in this figure).

Definition: *Let $\{x_1^*, x_2*, \ldots, x_n^*\}$ be the n ideal points for a committee of n individuals. Let N_R be the number of $x_i^* \geq x_m$, and N_L be the number of $x_i^* \leq x_m$. Then x_m is a median position iff $N_R \geq n/2$ and $N_L \geq n/2$.*

Theorem: *If x is a single-dimensional issue, and all voters have single-peaked preferences defined over x, then x_m, the median position, cannot lose under majority rule.*

Proof: Consider any $z \neq x_m$, say, $z < x_m$. Let R_m be the number of ideal points to the right of x_m. By definition of single-peaked preferences, all R_m voters with ideal points to the right of x_m prefer x_m to z. By definition of median position, $R_m \geq n/2$. Thus, the number of voters preferring x_m to z is at least $R_m \geq n/2$. x_m cannot lose to z under majority rule. Similarity, one can show that x_m cannot lose to any $z > x_m$. □

5.4 Majority rule and multidimensional issues

Single-peakedness is a form of homogeneity property of preference orderings (Riker, 1961, p. 908). People who have single-peaked preferences on an issue *agree* that the issue is one for which there is an optimum amount of the public good, and that the farther one is away from the optimum, the worse off one is. If quantities of defense expenditures were measured along the horizontal axis, then a preference ordering like the ordering in Figure 5.4 would obviously imply that Voter 1 is somewhat of a dove and Voter 5 a hawk, but a consensus of values would still exist with respect to the way in which the quantities of defense expenditures were ordered. The median voter theorem states that a consensus of this type (on a single-dimensional issue) is sufficient to ensure the existence of a majority rule equilibrium. During the Vietnam War, it was often said that some people favored *either* an immediate pullout or a massive expansion of effort to achieve total victory. Preferences of this type resemble Voter 2's preferences in Figure 5.3. Preference orderings such as these can lead to cycles. Note that the problem here may not be a lack of consensus on the way of viewing a single dimension of an issue, but on the dimensionality of the issue itself. The Vietnam War, for example, raised issues regarding both the U.S. military posture abroad and humanitarian concern for the death and destruction it wrought. One might have favored high expenditures to achieve the first, and a complete pullout to stop the second. These considerations raise, in turn, the question of the extent to which any issue can be viewed in a single dimension.

If all issues were unidimensional, multipeaked preferences of the type depicted in Figure 5.3 might be sufficiently unlikely so that cycling would not be much of a problem. In a multidimensional world, however, preferences of the type depicted in Table 5.1 seem quite plausible. Issues X, Y, and Z might, for example, be votes on whether to use a piece of land for a swimming pool, tennis courts, or a baseball diamond. Each voter could have single-peaked preferences on the amount to be spent on each activity, and a cycle would still appear over the issue of how the land should be used. The introduction of distributional considerations into a set of issues can, as already illustrated, also produce cycles.

A great deal of effort has been devoted to defining conditions under which majority rule does yield an equilibrium. Returning to Figure 5.4 we can see, somewhat trivially, that m emerges as an equilibrium because the other four voters are evenly "paired off" against one another regarding any move from m. This condition has been generalized by Plott (1967), who proved that a majority rule equilibrium exists if it is a maximum for one (and only one) individual, and the remaining even number of individuals can be divided into pairs whose interests are diametrically opposed; that is, whenever a proposal is altered so as to benefit a given individual A, a given individual B must be made worse off.

To see the intuition behind Plott's important result, consider first Figure 5.5. Let x_1 and x_2 be two issues, or two dimensions of a single issue. Let individual preferences be defined over x_1 and x_2, with point A the ideal point, the most preferred point in the x_1x_2 quadrant for individual A. If one envisages a third dimension, perpendicular

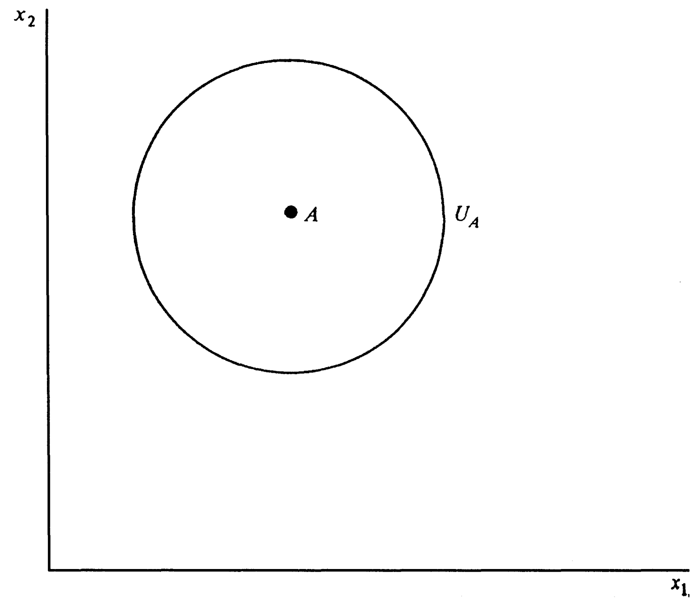

Figure 5.5. Outcome for a committee of one.

to the x_1x_2 plane, with utility measured in this third dimension, then point A is a projection of the peak of individual A's utility "mountain" onto the x_1x_2 plane. Pass a second plane through the mountain between its peak and floor and it will intersect the mountain in curves representing equal levels of utility. One such curve, drawn as a circle, is presented in Figure 5.5.

If we thought of individual A as a committee of one making choices using majority rule, then rather obviously and trivially she would choose point A. For her it is the dominant point in the x_1x_2 quadrant; that is, *it is a point that cannot lose to any other point*. What we seek to determine are the conditions for the existence of a dominant point under majority rule for committees larger than one.

Let B join A to form a committee of two. Under majority rule, any point that is off the contract curve, like D in Figure 5.6, can be defeated by a point on the contract curve, like E, using majority rule. Thus, no point off the contract curve can be a dominant point. At the same time, points like E on the contract curve cannot lose to other points on the contract curve like A and B. In a choice between A and E, voter A chooses A, B chooses E, and the result is a draw under majority rule. For a committee of two, the set of dominant points under majority rule is the contract curve. With circular indifference curves, the contract curve is the straight line segment joining A and B.

It should be clear from this example that dominance and Pareto optimality are closely related. Indeed, for E to be a dominant point, it must be in the Pareto set of every majority coalition one can construct, for were it not, there would exist some other point Z in the Pareto set for a majority coalition, which is Pareto preferred to E. This coalition will form and vote for Z over E.

Now consider a committee of three. Let C's ideal point be at C in Figure 5.7. The Pareto sets for each majority coalition are again the straight line segments joining

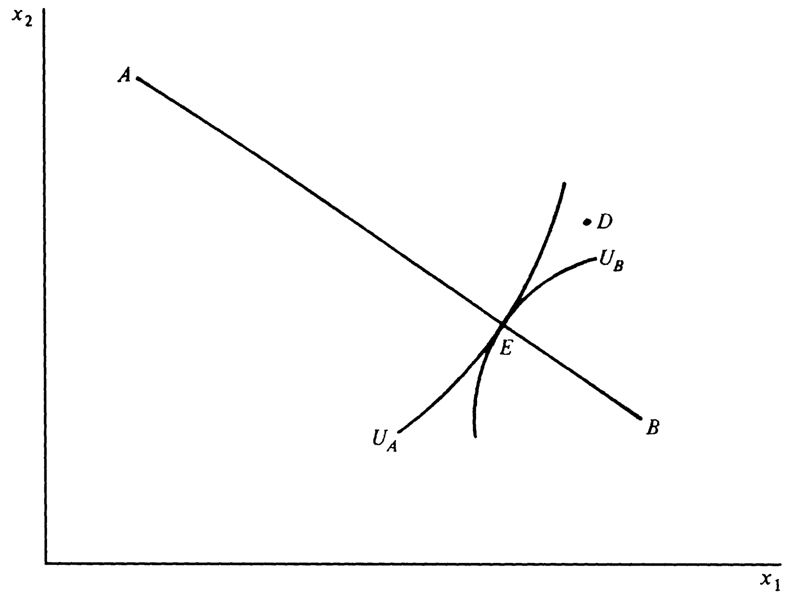

Figure 5.6. Outcomes for a committee of two.

each pair of ideal points, *AC*, *BC*, and *AB*. There is no point common to all three line segments, and thus no point is contained in all three Pareto sets. By the logic of the previous paragraph, there is no dominant point under majority rule. A point like *D* in $A - C$'s Pareto set lies outside of A–B's Pareto set. There thus exist points on *AB*, like *Z*, that can defeat *D*.

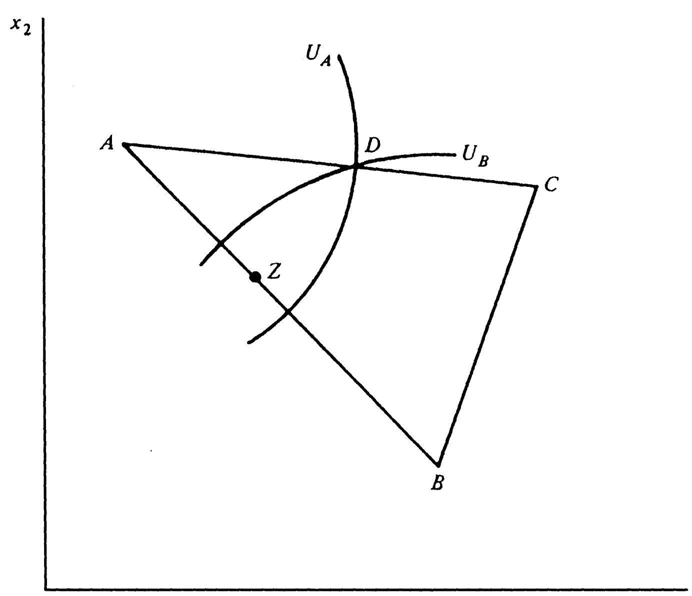

Figure 5.7. Cycling outcomes for a committee of three.

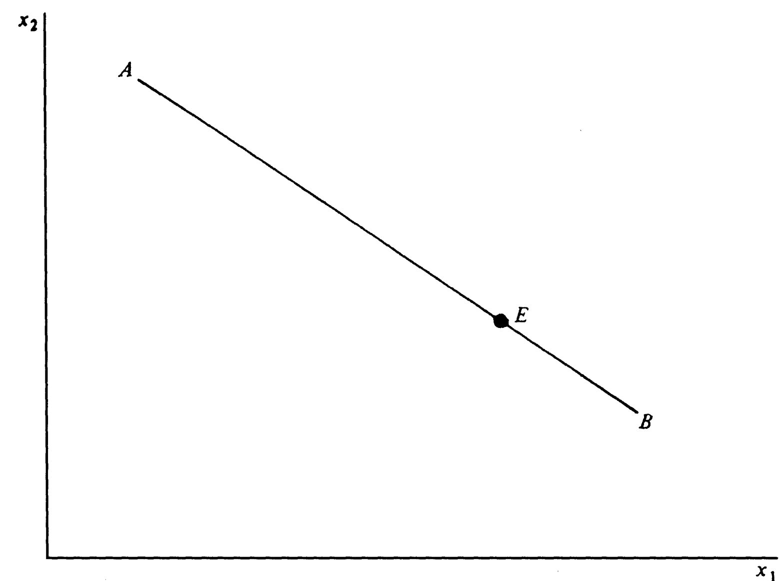

Figure 5.8. Equilibrium outcome for a committee of three.

The triangle ABC inclusive of its borders constitutes the Pareto set for the committee of three. Were the unanimity rule employed, the committee would be led to some point within ABC or on its boundary. Once there, the committee would be stuck, unable to move unanimously to another point. All points in and on ABC are potential equilibria. Under the majority rule, however, only the Pareto sets for the majority coalitions are relevant. There are three of them, but with no common point among them, no equilibrium exists.

The situation would be different if the third committee member's ideal point fell on the segment AB or its extension, say, at E (Figure 5.8). The three majority coalition Pareto sets are again the segments joining the three ideal points, AB, AE, and EB. However, now they have a point in common, E, and it is the dominant point under majority rule.

When the third committee member's ideal point falls on the ray connecting the other two members' ideal points, what was a multidimensional choice problem collapses into a single-dimensional choice problem. The committee must select a combination of x_1 and x_2 from along the ray through A and B. The conditions for the median voter theorem are applicable, and the committee choice is at the ideal point for the median voter, point E. Note also that the interests of the remaining committee members, A and B, are both diametrically opposed and "balanced" against one another as Plott's theorem requires for an equilibrium.

Now consider adding two more members to the committee. Obviously, if their ideal points were to fall along the ray through AB, an equilibrium would still exist. If one point were above and to the left of E and the other below and to the right, then E would remain the single dominant point under majority rule. But if both points fell outside of AB but were still on its extension, say, above and to the left of A, an equilibrium would still exist. In this case, it would be at A.

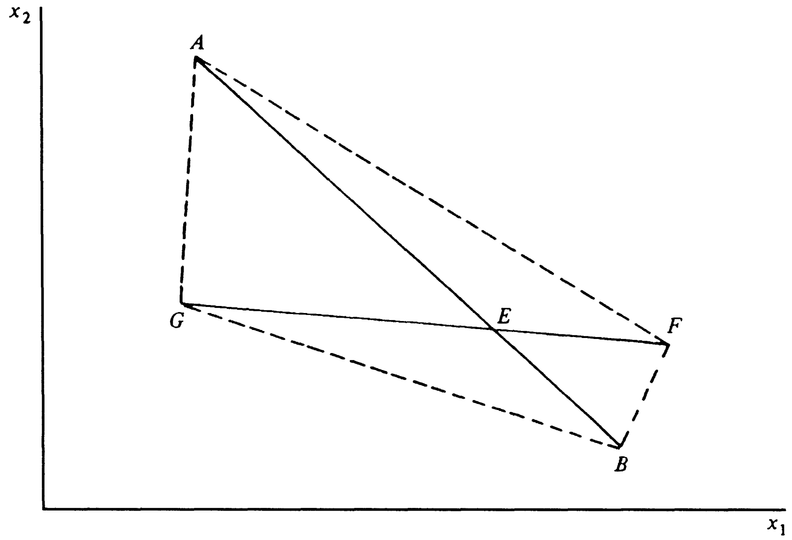

Figure 5.9. Outcome for a committee of five.

But the ideal points of the new members do not have to fall along *AB* extended for a dominant point to continue to exist. Suppose the two new committee members had ideal points falling on a line segment passing through E, but not coinciding with *AB*, say, like F and G in Figure 5.9. With a committee of five, three are needed to form a majority coalition. The Pareto sets for the majority coalitions are the triangles *AEF*, *AEG*, *GEB*, and *BEF*, and the line segments *AEB* and *GEF* (see Figure 5.9). These six Pareto sets have but one point in common, E, and it is the dominant point under majority rule. E remains the equilibrium because the two new members' interests are symmetrically positioned on opposite sides of E, and thus one's interests are balanced against the other's. As long as new committee members would continue to be added in pairs with ideal points on line segments passing through E, and on opposite sides of it, this balance would not be upset and E would remain the committee's equilibrium choice under majority rule.

The dominance of E in Figure 5.9 does not follow as it did in Figure 5.8 from a direct application of the median voter theorem. The issue space cannot be collapsed to a single-dimensional representation in Figure 5.9. But E is a median point in a more general sense. Pass any line through E, like *WW* in Figure 5.10, and there are three points on or to the left of (above) this line, as well as three points on or to the right of (below) it. A movement from E to the left will be opposed by a majority of the committee (*EBF*) as will a movement to the right (*EAG*). Since this is true for all possible lines that one can draw through E, all possible moves from E are blocked – hence its equilibrium nature. E satisfies the definition of a median point presented in Section 5.3*, with respect to the areas left and right of the line *WW* through E. The number of ideal points at or to the left of E is greater than $n/2$, as is the number at or to the right, where n, the committee size, in this case is 5. Since this property holds for every *WW* one can draw through E, E is a median point *in*

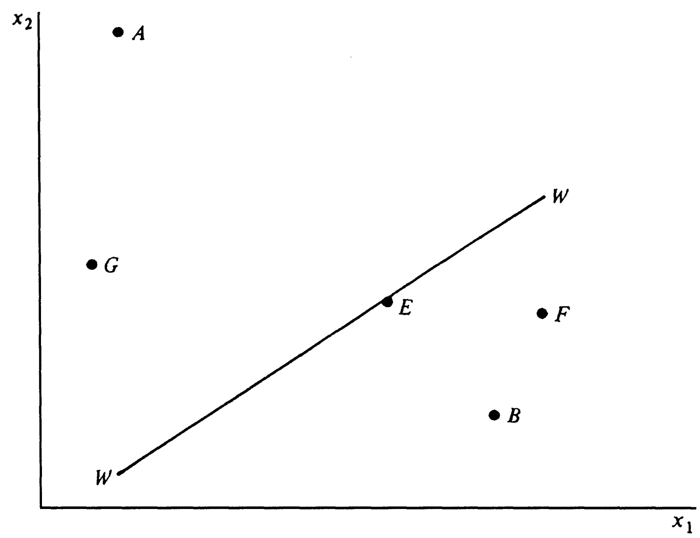

Figure 5.10.

all directions. The theorem – that the necessary and sufficient condition for E to be a dominant point under majority rule is that it be a median in all directions – is proved in the next section.

5.5* Proof of the median voter theorem – multidimensional case

This theorem was first proved by Davis, DeGroot, and Hinich (1972); we again follow Enelow and Hinich (1984, ch. 3).

We begin by generalizing the definitions of N_R and N_L. N_R is the number of ideal points to the right of (below) any line passing through E; N_L is the number of ideal points to the left of (above) this line. Continue to assume circular indifference curves.

Theorem: *E is a dominant point under majority rule iff $N_R \geq n/2$ and $N_L \geq n/2$ for all possible lines passing through E.*

Proof:

Sufficiency: Pick any point $Z \neq E$ (see Figure 5.11), and inquire whether Z might nevertheless defeat E under the simple majority rule. Draw *ZE*. Draw *WW* perpendicular to *ZE*. Given that all indifference curves are circles, E is closer to any ideal point to the right of (below) *WW* than is Z. N_R voters prefer E to Z. By assumption, $N_R \geq n/2$. E cannot lose to Z.

Necessity: We must show that if Z is a point not satisfying the $N_R \geq n/2$ and $N_L \geq n/2$ condition for some *WW* line drawn through it, then it cannot be a dominant point. Let Z and *WW* in Figure 5.12 be such that $N_R < n/2$. Then $N_L > n/2$. Now move *WW* parallel to its original position until it reaches some point Z' on the perpendicular to *WW* such that N_L' just satisfies the

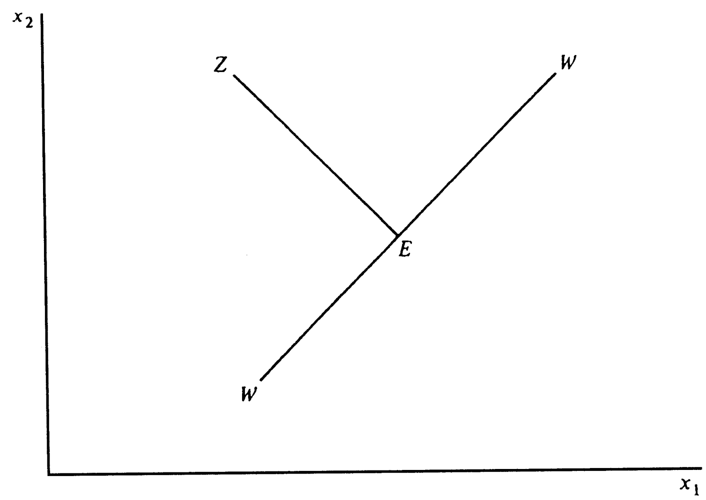

Figure 5.11.

condition $N_L' \leq n/2$ for the line $W'W'$ through Z'. Clearly, some point Z' satisfying this condition must eventually be reached. Now choose Z'' between Z and Z' on the line segment ZZ'. N_L'' defined with respect to the line through Z'' parallel to WW must satisfy $N_L'' > n/2$. But the N_L'' voters with ideal points to the left of $W''W''$ must all prefer Z'' to Z. Thus Z cannot be a dominant point. □

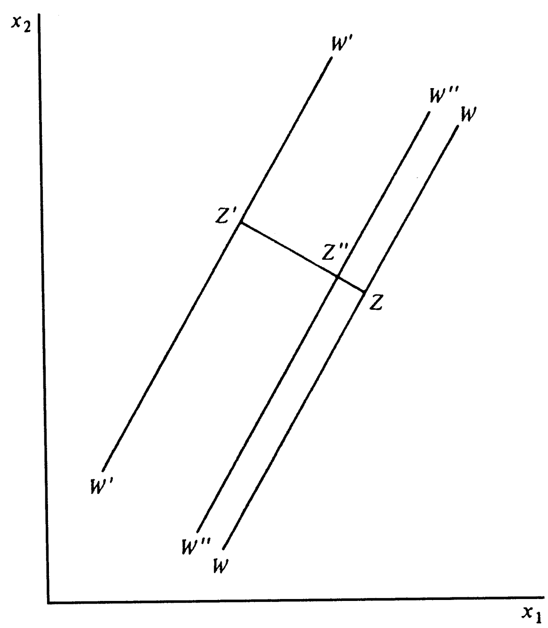

Figure 5.12.

5.6 Majority rule equilibria when preferences are not defined in spatial terms

So far, the results of this chapter regarding an equilibrium under majority rule have been derived in the context of a spatial model of choice. This is perhaps a natural way to approach choice questions for economists since they often analyze individual choices assuming utility functions defined over continuous variables and illustrate their results using geometry. But whether one views the results so far in a positive light (an equilibrium does exist under majority rule) or in a negative one (but only under very stringent assumptions), one might wonder how sensitive the results are to the formulation of the questions in spatial terms. Might better or worse results ensue if one abandoned the spatial context for examining majority rule? After all, voters do not typically think in spatial terms. These questions echo an attack on the public choice approach to politics levied by Stokes (1963), when public choice spatial models first began to intrude into the political science literature.

All of the major results concerning consumer behavior can be derived without the help of geometry or calculus, if one assumes that individual preferences satisfy certain basic rationality axioms (Newman, 1965). Since the theorems regarding consumer behavior derived from these axioms closely resemble those derived using calculus, one might suspect that the same will be true regarding collective decision functions like majority rule. And this suspicion is borne out.

The concept of an ideal point for an individual carries over directly into the axiomatic approach, if we assume that individual preferences satisfy the three axioms of reflexivity, completeness, and transitivity. Using R to denote the relationship "at least as good as," that is, either strict preference P or indifference I, then the axioms are

Reflexivity: For every element x in the set S, xRx.
Completeness: For every pair of elements x and y in the set S, $x \neq y$, either xRy, or yRx, or both.
Transitivity: For every triple x, y, and z in S, $(xRy \text{ and } yRz) \rightarrow (xRz)$.

If individual preferences satisfy these three axioms, then they define an *ordering* over the set of alternatives, S. The individual is assumed to be capable of ranking all of the alternatives in S, and the ideal point is then the alternative ranked highest, that is, the alternative preferred to all others.

Given that individual preferences are assumed to define an ordering, a natural way to approach the issue of whether an equilibrium exists under majority rule is to ask whether majority rule defines an ordering, in particular, to ask whether majority rule satisfies transitivity. If it does, then an alternative that beats (or at least ties) all others must exist in any set, and this is our dominant (equilibrium) outcome.

Majority rule does define an ordering over the set of alternatives S if individual preferences, in addition to satisfying the three axioms that define an ordering, also satisfy the extremal restriction axiom.[5]

[5] Sen and Pattanaik (1969). Other variants on this axiom (all equally restrictive) and on the basic theorem are discussed by Sen (1966, 1970a, chs. 10, 10*).

Extremal restriction: If for any ordered triple (x, y, z) there exists an individual i with preference ordering $x P_i y$ and $y P_i z$, then every individual j who prefers z to x $(z P_j x)$ must have preferences $z P_j y$ and $y P_j x$.

There are several things to observe about this axiom. First, although it does not require a spatial positioning of alternatives, it does require that individuals view alternatives in a particular way. Individuals must order issues x, y, z or z, y, z; they cannot order them y, x, z, for example.

Second, the condition does not require that all individuals have either the $x P_i y P_i z$ ordering or the $z P_j y P_j x$ ordering. The second part of the condition is only triggered if some individual prefers z to x. But no one may prefer z to x. All may either prefer x to z or be indifferent between them. If they are, then the theorem states that no cycle can occur.

Third, if one wants to think of the issues as ordered in a left-to-right way (x, y, z), then the condition resembles single-peakedness but is not equivalent to it. In particular, the condition allows for the preferences $x I_j z P_j y$ when the preferences $x P_i y P_i z$ are present. If y is the middle issue, then the preference ordering $x I_j z P_j y$ implies twin peaks at x and z. The condition does mandate, however, that the two peaks at x and z must be of equal altitude.

Although the extremal restriction avoids defining the issues in spatial terms, it is in other respects a severe constraint on the types of preference ordering people can have if majority rule is to satisfy transitivity. If a committee must decide whether a vacant lot is to be used to build a football field (x), tennis court (y), or a swimming pool (z), then some individuals may reasonably prefer football to tennis to swimming. But equally reasonably, others may prefer tennis to swimming to football. If both types of individuals are on the committee, however, the extremal restriction is violated and a voting cycle under majority rule may ensue. This theorem is proved in the next section.

5.7* Proof of extremal restriction – majority rule theorem

Theorem: *Majority rule defines an ordering over any triple (x, y, z) iff all possible sets of individual preferences satisfy extremal restriction.*

The proof follows Sen (1970a, pp. 179–81).

Sufficiency: The most interesting cases involve those in which at least one voter has preferences:

1. $x P_i y P_i z$.

In addition to voters of type 1, the extremal restriction allows there to be voters with the following four sets of preference orderings:[6]

2. $z P_j y P_j x$
3. $y P_j z I_j x$

[6] In fact, it allows for more than these four, but the others are eliminated once there is one voter for whom $z P x$.

4. zI_jxP_jy
5. zI_jxI_jy.

Voters of type 5 can be assumed to abstain, and shall be ignored hereafter. Now assume that the theorem does not hold; that is, assume the existence of a forward cycle

$$xRy,\ yRz,\ \text{and } zRx,$$

where the unsubscripted R implies the social ordering under majority rule. Call $N(zP_ix)$ the number of individuals who prefer z to x:

$$(zRx) \rightarrow [N(zP_ix) \geq N(xP_iz)]. \tag{5.1}$$

By assumption, at least one individual has the ordering xP_iyP_iz. Thus,

$$N(xP_iz) \geq 1 \tag{5.2}$$

and from (5.1)

$$N(zP_ix) \geq 1.^7 \tag{5.3}$$

Call N_1 the number of individuals with preferences as given in (1) above, N_2 as in (2), and so on.

$$(xRy) \rightarrow (N_1 + N_4 \geq N_2 + N_3) \rightarrow [N_4 \geq (N_2 - N_1) + N_3] \tag{5.4}$$

$$(yRz) \rightarrow (N_1 + N_3 \geq N_2 + N_4) \rightarrow [N_3 \geq (N_2 - N_1) + N_4] \tag{5.5}$$

$$(zRx) \rightarrow (N_2 \geq N_1). \tag{5.6}$$

For both (5.4) and (5.5) to hold,

$$N_2 = N_1 \tag{5.7}$$

and thus

$$N_3 = N_4. \tag{5.8}$$

But then

$$(N_2 + N_3 \geq N_1 + N_4) \rightarrow (yRx) \tag{5.9}$$

$$(N_2 + N_4 \geq N_1 + N_3) \rightarrow (zRy) \tag{5.10}$$

$$(N_1 \geq N_2) \rightarrow (xRz). \tag{5.11}$$

However, (5.9) through (5.11) imply a backward cycle. Thus, if extremal restriction is satisfied, a forward cycle can exist only in the special case when a backward cycle does. A cycle ensues because society is indifferent among all three issues. The number of voters preferring x to y equals the number preferring y to x, the number preferring y to z equals the number preferring z to y, and the number preferring x to z equals the number preferring z to x.

[7] Conditions (5.2) and (5.3) ensure that the only preference orderings in the committee that can satisfy the extremal restriction axiom are among the five types given above.

If one assumes the theorem is violated by a backward cycle, an analogous argument demonstrates that extremal restriction also implies a forward cycle.

Necessity: We must show that violation of the extremal restriction axiom can lead to intransitive social preferences under the majority rule.

Assume one i with

$$xP_i yP_i z. \tag{5.12}$$

Extremal restriction is violated if one j has the ordering

$$zP_j x \text{ and } zP_j y \text{ and } xR_j y \tag{5.13}$$

or the ordering

$$zP_j x \text{ and } yP_j x \text{ and } yR_j z. \tag{5.14}$$

Assume (5.12) and (5.13) hold. Then under majority rule

$$xPyIzIx,$$

which violates transitivity.

Next assume (5.12) and (5.14) hold. Then under majority rule

$$xIyPzIx,$$

which is again in violation of the transitivity axiom. When the extremal condition is not satisfied, majority rule may be incapable of producing a complete ordering over all alternatives.

5.8 Restrictions on preferences, on the nature and number of issues, and on the choice of voting rule that can induce equilibria

5.8.1 *Preference homogeneity*

For the reader who is unfamiliar with the public choice literature, the results on majority rule equilibrium must seem both surprising and disconcerting. Can the most frequently employed voting rule really produce the kind of inconsistency implied by its violation of the transitivity property? Are the types of preferences needed to bring about an equilibrium under majority rule really as unlikely to arise naturally as the preceeding theorems suggest?

Unfortunately, the answers to these questions appear to be "yes." This is nicely illustrated in Kramer's (1973) generalization of the single-peakedness condition to more than one dimension. Kramer's theorem is particularly revealing to economists because he explores voter choices in the familiar environment of budget constraint lines and convex indifference curves.

In Figure 5.13, let x_1 and x_2 represent the quantities of two public goods, or two attributes of a single public good. BB is the budget constraint line for the committee. All points on or within BB are feasible alternatives. Let U_1^A and U_2^A be two indifference curves for individual A. A's preferences over the triple (x, y, z)

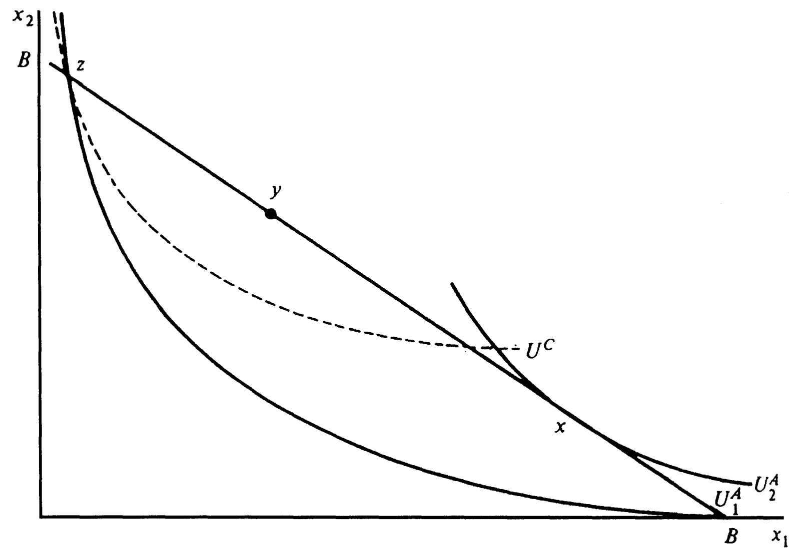

Figure 5.13. Possible cycles with normal indifference curves.

are xP_AyP_Az. Let C have the dotted indifference curve U^C. C's preferences over (x, y, z) are yP_CzP_Cx. The extremal restriction defined in Section 5.6 is violated. With individuals like A and C on the committee, majority rule may produce a cycle over triples like (x, y, z) selected from the feasible set. But there is nothing unusual about A and C's indifference curves other than that they intersect. When can we be certain that we avoid all preference orderings that violate extremal restriction over the feasible set? Only when all individuals have identical indifference maps, or as Kramer (1973, p. 295) puts it, when there is "complete unanimity of individual preference orderings."[8]

And so we return to a unanimity condition. If what we seek is a voting rule to reveal individual preferences on public goods, the options would appear to be as follows. A unanimity rule might be selected that possibly requires an infinite number of redefinitions of the issue until one that benefited all citizens was reached. Although each redefinition might, in turn, be defeated until a point on the Pareto-possibility frontier had been reached, once attained, no other proposal could command a unanimous vote against it, and the process would come to a halt. The number of times an issue must be redefined before a passing majority is reached can be reduced by reducing the size of the majority required to pass an issue. Although this "speeds up" the process of obtaining the *first* passing majority, it slows down, perhaps indefinitely, the process of reaching the *last* passing majority, that is, the one that beats all others. For under a less-than-unanimity rule, some voters are made worse off. This is equivalent to a redistribution from the opponents of a measure to its proponents. As with any redistribution measure, it is generally possible to redefine an issue transferring the benefits among a few individuals and to obtain a new winning coalition. The Plott "perfect balance" condition ensures an equilibrium under majority rule by imposing

[8] Were we to allow pairwise comparisons among all points along *BB* and exclude all points within *BB* from consideration, then convex utility functions would imply single-peaked preferences along the one dimension *BB* defines, and the median voter theorem would apply. Allow points interior to *BB* to be chosen under majority rule, or add a third dimension to the issue set and this escape hatch is closed, however.

a form of severe symmetry assumption on the distribution of preferences that ensures that any redefinition of an issue always involves symmetric and offsetting redistributions of the benefits. The same counterbalancing of interests is contained in the median-in-all-directions condition, while extremal restriction also tends to limit the contest to those with strictly opposing interests (for example, xP_iyP_iz types against zP_jyP_jx types). The Kramer "identical utility functions" condition removes all conflict, and thereby eliminates all questions of redistribution.

The redistributive characteristics of less-than-unanimity rules explain the similarities between the proofs and conditions for a majority rule equilibrium, and those establishing a social welfare function (or the impossibilities thereof). Both flounder on their inability to choose among Pareto-preferred points, that is, to handle the question of redistribution (see Sen, 1970a, chs. 5 and 5*).

These theorems all establish the *possibility* of a cycle when their restrictive conditions are not met. They do not establish the inevitability of a cycle. As Kramer (1973) notes, the existence of a majority with identical preferences is sufficient to ensure a majority rule equilibrium regardless of the preferences of all other voters (see also Buchanan, 1954a). More generally, we might wish to inquire as to how often in practice a set of preferences arises that leads to a cycle.

A large number of studies have computed the probabilities of cycles using simulation techniques. When no special restrictions are placed on the types of preference orderings individuals may have, the probability of a cycle is high, and approaches one as the number of alternatives increases.[9] We have noted that a cycle cannot occur if a majority of voters have identical preferences. Thus, we might expect that as various homogeneity assumptions are made about voter preferences, the probability of a cycle decreases. And this is so. Niemi (1969) and Tullock and Campbell (1970) found that the probability of a cycle declines as the number of single-peaked preferences increases. Williamson and Sargent (1967), and Gehrlein and Fishburn (1976a) found that the probability of cycles declines with the proportion of the population having the same preferences,[10] and similarly Kuga and Nagatani (1974) have discovered that it increases with the number of pairs of voters whose interests are in conflict. These results suggest that the probability of a cycle under majority rule would be low if the collective choice process were restricted to movements from off the contract curve to points on it – that is, the kinds of decisions the unanimity rule might be able to handle – where voter interests tend to coincide.

5.8.2 *Homogeneous preferences and qualified majority rules*

The results reviewed in Section 5.8.1 indicate that the probability of cycles under the simple majority rule falls as voter preferences become more homogeneous. The probability of a cycle can also be reduced by increasing the majority required to defeat the status quo.

To see this, consider Figure 5.14a. A community must decide the quantities of two public goods, x_1 and x_2, as before. The citizens' ideal points are uniformly

[9] Garman and Kamien (1968), Niemi and Weisberg (1968), DeMeyer and Plott (1970), Gehrlein and Fishburn (1976b). This literature is reviewed in Niemi (1969), Riker and Ordeshook (1973, pp. 94–7), and Plott (1976).

[10] See also Abrams (1976) and Fishburn and Gehrlein (1980).

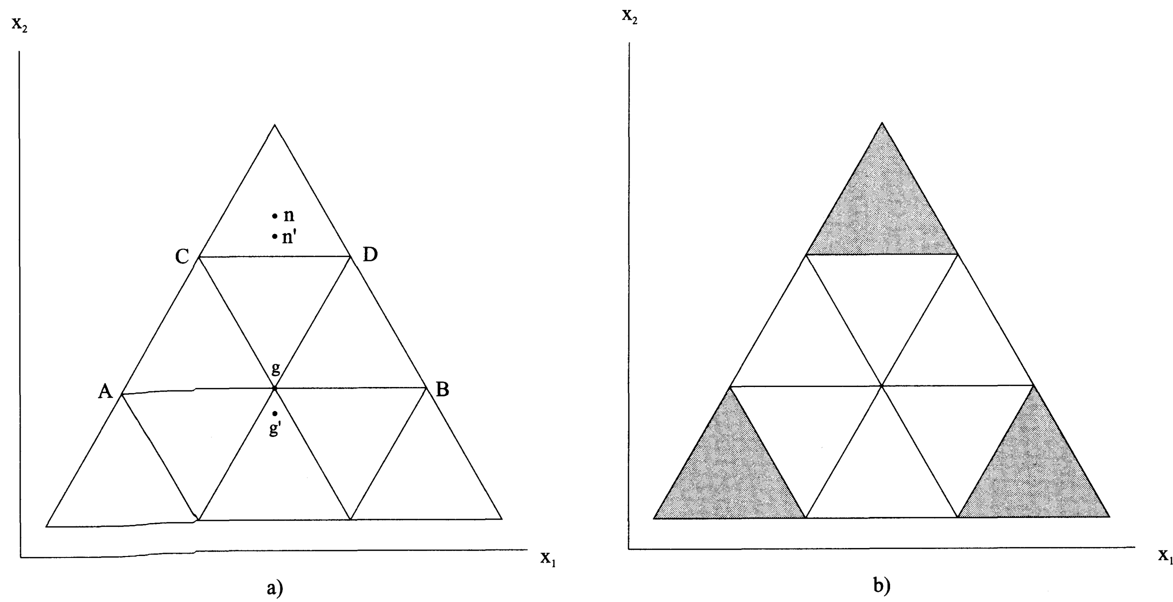

Figure 5.14. Equilibria with different qualified majority rules.

distributed over an area that forms an equilateral triangle. Each point in the triangle represents one voter's ideal point. The lines through the triangle divide it into nine smaller triangles of equal areas. No point in the large triangle satisfies Plott's perfect balance condition, and thus there is no equilibrium under the simple majority rule. For example, point g, the center of gravity of the large triangle, would lose to a point slightly below it like g'. There are five small triangles below the horizontal line $\overline{AB}$ through g, and only four above it. Thus, five ninths of the citizens prefer points below g to it, and some point like g' can win a majority against g.

On the other hand, *every* point in the large triangle is an equilibrium under the unanimity rule. The large triangle constitutes the Pareto set and once a proposal in the Pareto set has been accepted as the status quo, any attempt to move from it will be vetoed. Intuitively one expects that the set of points that are possible equilibria shrinks as the majority required to displace the status quo is reduced until it becomes the null set. And this intuition is correct. Under an 89 percent required majority, for example, point n will lose against any point slightly below it like n', since 89 percent of the ideal points lie below line $\overline{CD}$, and thus more than 89 percent of the community prefers n' to n. None of the points in the three shaded triangles in Figure 5.14b is an equilibrium under an 89 percent required majority, since for each such point another can be found within the unshaded region that can defeat it. None of the points in the six unshaded triangles can lose to any other point under an 89 percent majority rule.

The smallest majority that produces an equilibrium outcome in this situation is a five-ninths majority. There are five small triangles on one side of each line drawn through g, and four on the other. If more than five ninths of the population must vote for a proposal for it to defeat g, then the citizens with ideal points located in the four triangles can block any proposal by the other citizens to replace g with a point in the five-triangle space. Any other line drawn through g as, say, a vertical line, divides the large triangle into two areas, each containing less than five-ninths of the population. Thus no point can defeat g under a five-ninths majority rule. It is the unique stable equilibrium in this situation.

This example raises the question of whether it is possible to determine for different situations the minimum qualified majority that guarantees the existence of an equilibrium. This question was first addressed by Black (1948b). Under the assumption that all individuals have convex preferences defined over an n-dimensional issue space, Greenberg (1979) proved that m^*, the required majority to guarantee the existence of at least one equilibrium point in the issue space, must satisfy the following condition:

$$m^* \geq \frac{n}{(n+1)}. \tag{5.15}$$

With $n = 1$, $m^* = 0.5$ and (5.15) merely restates the median voter theorem. With convex preferences defined over a single-dimensional issue space, requiring one vote more than a 50 percent majority suffices to guarantee the existence of an equilibrium outcome. Equation (5.15) implies, however, that m^* continues to rise and approaches unanimity as the number of dimensions in the issues space rises.

In an important further development, Caplin and Nalebuff (1988) have shown that m^* can be significantly lowered by placing restrictions on *both* the preferences of members of the community and the distribution of their ideal points. With a two-dimensional issue space each individual's utility is as depicted in Figure 5.5, namely, she has a most preferred combination of x_1 and x_2, and her utility falls off as the chosen combination moves away from this ideal point. If utility were depicted along a third axis perpendicular to the page, it would take the shape of a cone or mountain with its peak at the ideal point A. Now imagine placing each member of the committee's utility mountain on Figure 5.5, and that the aggregation of all of these mountains is itself a mountain with a single peak somewhere within the $x_1\, x_2$ quadrant. Given these assumptions about individual preferences and the distribution of their ideal points, Caplin and Nalebuff prove that m^* must satisfy the following condition:

$$m^* \geq 1 - \left(\frac{n}{n+1}\right)^n. \tag{5.16}$$

Once again when $n = 1$, $m^* = 0.5$. When $n = 2$, $m^* = 5/9$ as in the example above, and m^* continues to increase with n, reaching a maximum of less than 64 percent, since the limit of $(n/(n+1))^n$ as n approaches infinity is $1/e$, and $1/e < 0.368$. A 64 percent majority suffices to ensure the existence of at least one point in any n-dimensional issue space that cannot lose to any other point, even when n is infinitely large. Preferences of the type needed to establish (5.16) seem quite reasonable *if* voting is on quantities of public goods, and the tax formulas for financing the public goods are predetermined.[11] The assumption that the density function of the voters' ideal points be concave is much stronger, and imposes a degree of *social consensus* on the community (the community is not divided into clusters of different voters each favoring combinations of public good quantities that differ radically from one another). Assuming a generalized single-peakedness in more than one direction plus a degree of social consensus suffices to eliminate the possibility of cycles, if we are willing to abandon the simple majority rule for a 64 percent qualified majority.[12]

This result of Caplin and Nalebuff requires that we reconsider the question of the optimal majority for a voting rule discussed in Chapter 4. In Figures 4.4 and 4.5, we depicted decision-making costs rising continuously from a required majority of 0.5. Such an assumption might be reasonable, if we thought of the process as a search for new tax/quantity combinations that allow us to add one person at a time to an ever-growing coalition that favored each new proposal. If, however, we think of the community's task as that of choosing a combination of several public good quantities or attributes, a more reasonable assumption may be that each new proposal drops some members from the previous winning coalition and adds new

[11] Individual preferences do not have to yield circular indifference curves as in Figure 5.5; the preferences need only be single-peaked in the n-dimensional issue space. The reader is referred to Caplin and Nalebuff (1988, pp. 790–2) for a full statement of the assumptions needed for the proof.

[12] The assumption that the distribution of voter ideal points is concave is relaxed to allow for log-concavity in Caplin and Nalebuff (1991), where a *mean* voter theorem in an n-dimensional issue space is proved.

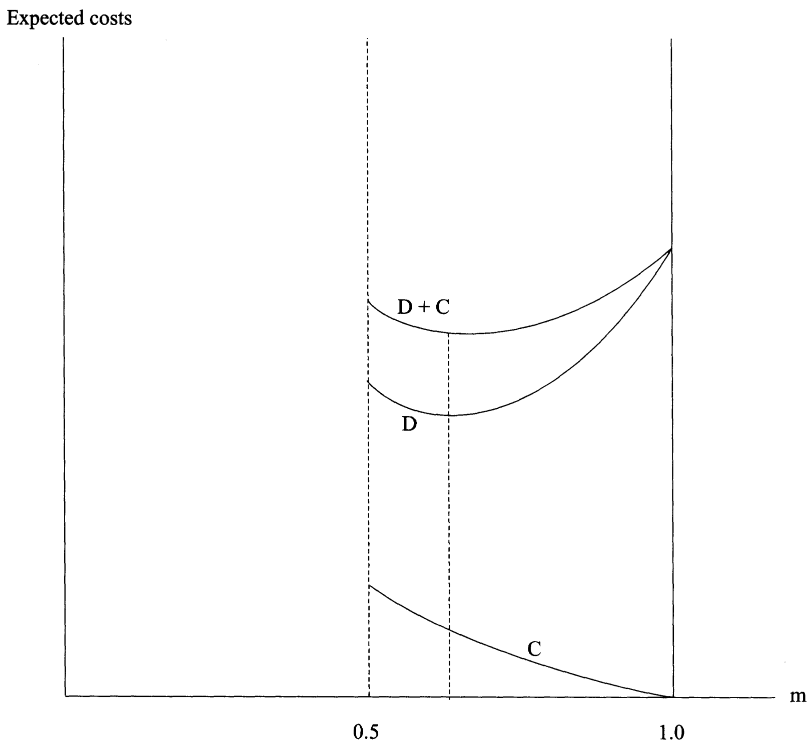

Figure 5.15. The optimal majority with cycling.

ones. We have seen how such changes in coalition composition can generate cycles. Caplin and Nalebuff's theorem suggests that in this sort of environment, decision-making costs may actually *fall* as the majority required to pass an issue rises from 0.5 until cycles are no longer possible. The D-curve would now have a U-shape, and whether it is discontinuous at an m of 0.5 would be irrelevant, as the D-curve would reach a minimum to the right of 0.5. With the bottom of the U somewhere around a majority of 0.64, the combined $C + D$ costs would then reach a minimum slightly to the right of the bottom of the U in D, and something like a two-thirds qualified majority would minimize the sum of decision-making costs and the external costs of collective decisions (see Figure 5.15).[13]

5.8.3 *The relationship between numbers of issues and alternatives and the required majority*

In a spatial world where one chooses different combinations of public goods quantities, the set of possible alternatives is infinite. One way to eliminate the possibility of cycles beyond raising the majority required to choose an alternative is to limit the number of alternatives in the issue set. This result is nicely illustrated in a theorem of James Weber (1993).

[13] Coggins and Perali (1998) suggest that the Venetians understood the advantage of using a 64 percent majority rule already in the thirteenth century as is revealed by their choice of rules for choosing the Doge.

Theorem: *Let N be the number of voters, $N \geq 2$, A the number of alternatives, $A \geq 2$, and M the number of voters required to select an alternative, $(N/2) < M \leq N-1$. Then there exists at least one set of individual preference orderings that leads to a cycle, if and only if (5.17) is satisfied:*

$$\left[N \geq \left(\frac{A}{A-1}\right)M\right] \longleftrightarrow \left[M \leq \left(\frac{A-1}{A}\right)N\right] \longleftrightarrow \left[A \geq \left(\frac{N}{N-M}\right)\right]. \tag{5.17}$$

It is clear from the left-most inequality in (5.17) that the likelihood that the condition for a possible cycle is satisfied is greater, the greater N is for a given A and M. The right-most inequality in (5.17) reveals that the likelihood that the condition for a possible cycle is fulfilled is greater, the greater the number of alternatives is holding N and M constant. The middle inequality is related to the theorem of Caplin and Nalebuff. For any given numbers of alternatives A, and committee members N, a required majority to pass an issue exists, which is sufficiently high to eliminate the possibility of all cycles. For very large N and three alternatives, this majority is two-thirds; with six alternatives it is five-sixths; and so on. Given that the Caplin and Nalebuff result holds effectively for an infinite number of alternatives and very large electorates, we see that the cost of not placing restrictions on the shapes of committee members' preferences and their distribution, as Weber's theorem does not, is to require very high majorities to eliminate cycles, even with fairly small numbers of issue alternatives.

5.9 Logrolling

When faced with a simple binary choice between X and $\sim X$ under majority rule, an individual's obvious best (dominant) strategy is to state honestly his preference for X or $\sim X$. Majority rule records only these ordinal preferences for each individual on the issue pair. The condition for the Pareto optimality of the supply of public goods requires information on the relative intensity of individual preferences; however, the marginal rates of substitution of public for private goods must sum to the ratio of their prices. Since this information is not directly gathered under majority rule, it is not particularly surprising that the outcomes under majority rule may not satisfy the Pareto-optimality condition.

The Pareto-optimal allocation of private goods also requires information on individual preference intensities, but this information is elicited by the "voting" process for private goods as individuals selfishly engage in the exchange of goods and services to maximize their own utilities. But with voting on public issues, each individual is constrained to cast but one vote for or against a given issue – unless, of course, one allows individuals to exchange votes.

The buying and selling of votes by individual citizens is outlawed in all democratic countries. That such laws exist and are occasionally violated suggests that individual

Table 5.2. *Vote trading example*

	Issues	
Voters	X	Y
A	−2	−2
B	5	−2
C	−2	5

intensities of preference regarding the value of a vote do differ. Although buying and selling votes is also prohibited in parliamentary bodies, the more informal process – "you vote for my pet issue and I'll vote for yours" – is difficult to police. Exchanges of this sort have occurred in the U.S. Congress for as long as it has been in existence. That they do exist, in spite of a certain moral stigma to their use, has two implications. Intensities of preference on issues must differ across congressmen. The assumption that congressmen's actions can be explained as the pursuit of self-interest is buttressed. The natural inclination to engage in trade, "to truck and barter," as Adam Smith called it, seems to carry over to the parliamentary behavior of elected representatives.

To understand the process, consider Table 5.2. Each column gives the utility changes to three voters from an issue's passage; defeat produces no change. If each is decided separately by majority rule, both fail. Voters *B* and *C* have much to gain from *X* and *Y*'s passage, however, and can achieve this if *B* votes for *Y* in exchange for *C*'s vote for *X*. Both issues now pass to *B* and *C*'s mutual benefit.

The existence of beneficial trades requires a nonuniform distribution of intensities. Change the two 5s to 2s and *B* and *C* gain nothing by trading. This equal intensity condition is often invoked in arguments in favor of simple (without trading) majority rule, and are taken up in Chapter 6 when we consider the normative case for majority rule.

The trade between *B* and *C* can be said to have improved the welfare of the community of three voters if the numbers in Table 5.2 are treated as cardinal, interpersonally comparable utilities. Without trading, the majority tyrannizes over the relatively more intense minority on each issue. Through vote trading, these minorities express the intensity of their preferences, just as trading in private goods does, and improve the total welfare change of the community. With trading there is a net gain of 2 for the community.

An obvious condition for an improvement in community welfare through the changes in outcomes that vote trading brings about is that the cumulative potential utility changes for the (losing) minority members exceed the cumulative potential utility changes for the winning majority members on the issues involved. Change the 5s to 3s or the −2s of *A* to −4s, and the same trades emerge as before, since the pattern of trades depends only on the *relative* intensities of preferences of the voters. The sum of utilities for the community with trading is then negative, however. An exchange of votes increases the likelihood of the participants winning on their relatively more important issues. It *tends*, therefore, to increase their realized gains.

Table 5.3. *Trading possibilities*

			Utilities		
Winning pair	Losing pair	Trading voters	A	B	C
X, Y	~X, ~Y	B and C	−4	3	3
X, ~Y	X, Y	A and B	−2	5	−2
~X, ~Y	X, ~Y	A and C	0	0	0

These increases *can* increase the utility gain for the entire community. However, trading also imposes externalities (utility losses) on the nontraders who would have been better off in the absence of trading,[14] and, if these are large, they can outweigh the gains to the traders, lowering the community's net welfare. Critics of logrolling have typically envisaged situations such as these. They assume that the cumulative potential gains of the majority exceed those of the minority. Vote trading that reverses some of the outcomes of simple majority rule lowers collective welfare when this is true.

Tullock's (1959) argument that majority rule *with trading* can lead to too much government spending is of this type. Let A, B, and C be three farmers; and X be a road of use to only farmer B, and Y a road of use to only C. If the gross gains to a farmer from the access road are 7 and the cost is 6, which is shared equally, we have the figures of Table 5.3. With these costs and benefits, total welfare is improved by logrolling. But a bill promising a gross gain of 5 at a cost of 6, equally shared, also passes. Such a bill lowers community welfare by excessively constructing new roads, roads whose total benefits are less than their total costs. Again, the problem arises because majority rule can involve allocation and redistribution at the same time. The two bills involve both the construction of roads with gross benefits of 5 and costs of 6, and the redistribution of wealth from A to B and C; the latter can be sufficient to pass the bills.

An important difference separating logrolling's critics and proponents is their views as to whether voting is a positive- or negative- (at best zero-) sum game. If the latter, the game is obviously bad to begin with, and anything that improves its efficiency can only worsen the final outcome. The numerical examples that Riker and Brams (1973) present in their attack on logrolling are all examples of this type, and the examples they cite of tariff bills, tax loopholes, and pork barrel public works are all illustrations of bills for which a minority benefits, largely from the redistributive aspects of the bill, and the accumulative losses of the majority can be expected to be large.[15] The worst examples of logrolling cited in the literature are always issues of this type in which private or local public goods are added to the agenda for redistribution purposes to be financed out of public budgets at a higher level of aggregation than is appropriate (Schwartz, 1975). The best the community can hope for is the defeat of all of these issues. Riker and Brams (1973) logically recommend reforms to eliminate logrolling opportunities.

[14] See Taylor (1971, p. 344) and Riker and Brams (1973).

[15] See also Schattschneider (1935), McConnell (1966), and Lowi (1969).

A private good or a very local public good of course will be of great interest to a few and of little interest to the majority. The conditions necessary for logrolling are likely to be satisfied, therefore, through the incorporation of these goods into the community's agenda. But preference intensities can also vary considerably across individuals on what are truly pure public goods – for example, defense, education, and the environment. On issues such as these, vote trading can be a superior way for revealing individual preference intensities over the public goods.

One of the most positive and influential discussions of vote trading's potential was presented by Coleman (1966b). He depicted the members of the committee or legislature as entering into logrolling agreements on all public good issues. Each voter forms agreements to swap votes with other voters of the type described above. Each voter increases his ability to *control* those *events* (issues) about which he feels most intense in exchange for a loss of control over those events about which he cares little. A form of ex ante Pareto optimum is reached in which no voter feels he can increase his expected utility by agreeing to exchange another vote. This equilibrium is the optimum of Coleman's social welfare function.

Unfortunately, whatever potential a vote trading process has for revealing relative intensities of preference, and thereby improving the allocation of public goods, may go unrealized, because the trading process may not produce stable coalitions nor be free from strategic misrepresentation of preferences. When vote trades are parts of only informal agreements and take place in sequence, voters are motivated both to misstate their preferences at the time an agreement is formed and to violate the agreement after it is made. A voter who would benefit from X might pretend to oppose it and secure support for some other issues he favors in "exchange" for his positive vote for X. If successful, he wins on both X and the other issue. But the other "trader" might be bluffing, too, and the end results of trading become indeterminate (Mueller, 1967).

Even when bluffing is not a problem, cheating may be. When issues are taken up seriatim, there is an obvious and strong incentive for the second trader to renege on his part of the bargain. This incentive must be present, since the same preference orderings that produce a logrolling situation imply a potential voting cycle. Consider again the example in Table 5.3. In addition to X and Y with payoffs as in Table 5.3, we have the issues $\sim X$ and $\sim Y$ that "win" if X and Y fail. Both have payoffs for the three voters (O, O, O). Thus, four combinations of issues might result from the voting process: (X, Y), $(\sim X, Y)$, $(X, \sim Y)$, and $(\sim X, \sim Y)$. The committee must choose one of these four combinations. If we envisage voting as taking place on the issue pairs, then a cycle exists over the three pairs $(\sim X, \sim Y)$, (X, Y), $(X, \sim Y)$. In terms of the vote-trading process, the existence of this cycle implies that no stable trading agreements may be possible. We have seen that a trade between B and C to produce (X, Y) would make them both better off than the no-trade outcome $(\sim X, \sim Y)$ (see Table 5.3). But A can improve her position by offering to vote for X if B refrains from voting for Y. Thus, (X, Y) can be beaten (blocked) by $(X, \sim Y)$. But C can then offer A the option of no loss of utility if they both agree to vote sincerely and reestablish the victory of $(\sim X, \sim Y)$. From here the trading cycle can begin again. Moreover, the only condition under which a potential logrolling situation is certain not to create the potential for a cycle is when a unanimity rule is imposed (Bernholz, 1973). Allowing for individual intensity differences as in a logrolling process does not allow us to escape the cycling problem. On the contrary, the existence of the one implies the presence of the other, as we shall now demonstrate.

5.10* Logrolling and cycling

We illustrate the theorem following Bernholz (1973) with the simple example of the previous section. The key assumption is that each voter i has a well-defined preference ordering, which satisfies the following independence condition over the relevant issues.

Independent issues: If $XP_i \sim X$, then $(XY)P_i(\sim XY)$.
All voters vote sincerely at each juncture.

Definition: *A logrolling situation exists if*

$$\sim XRX \tag{5.18}$$

$$\sim YRY \tag{5.19}$$

$$XYP \sim X \sim Y, \tag{5.20}$$

where R and P are the social preference orderings defined by whatever voting rule is being used. In a pairwise vote, $\sim X$ defeats X and $\sim Y$ defeats Y. But the pair XY can defeat $\sim X \sim Y$.

Theorem: *The existence of a logrolling situation implies intransitive social preferences. The existence of a transitive social preference ordering implies the absence of a logrolling situation.*

Proof of First Proposition: Assume a logrolling situation exists [i.e., (5.18), (5.19), and (5.20) hold]. Then winning coalitions h must exist (i.e., majority coalitions under majority rule) for which

$$\sim XR_h X \tag{5.21}$$

$$\sim YR_h Y \tag{5.22}$$

$$XYP_h \sim X \sim Y. \tag{5.23}$$

From (5.21) and (5.22) and the independent issues assumption,

$$\sim X \sim YR_h X \sim Y \tag{5.24}$$

$$X \sim YR_h XY. \tag{5.25}$$

Since each respective h is a winning coalition,

$$\sim X \sim YRX \sim Y \tag{5.26}$$

$$X \sim YRXY. \tag{5.27}$$

Combining (5.20), (5.26), and (5.27), we have

$$\sim X \sim YRX \sim YRXYP \sim X \sim Y. \qquad (5.28)$$

The existence of a logrolling situation implies intransitive social preferences. □

Proof of Second Proposition: We assume the first part of a logrolling situation exists and demonstrate that transitive social preferences imply the absence of the second part (5.20); that is, assume

$$\sim XRX \qquad (5.18)$$

$$\sim YRY. \qquad (5.19)$$

This implies

$$\sim XR_h X \qquad (5.29)$$

$$\sim YR_h Y. \qquad (5.30)$$

By the independent issue assumption,

$$\sim XYR_h XY \qquad (5.31)$$

$$\sim X \sim YR_h \sim XY. \qquad (5.32)$$

Since each h is a winning coalition,

$$\sim XYRXY \qquad (5.33)$$

$$\sim X \sim YR \sim XY. \qquad (5.34)$$

But then

$$\sim X \sim YR \sim XYRXY. \qquad (5.35)$$

If the social preferences are transititve, then $\sim X \sim YRXY$, and the last part of the definition of a logrolling situation is not satisfied. The existence of transitive social preferences implies the absence of a logrolling situation. □

5.11 Testing for logrolling

Claims of "horse trading" to create majority coalitions and select cabinets in Europe, and to pass legislation in the United States are as old as democracy in these countries.[16] But because vote trading takes place in "smoke filled rooms" out of the public's eye, it is often difficult to verify that it has in fact taken place and to identify the traders. Does vote trading occur on all legislation in the U.S. Congress, some, or none? If it occurs only some of the time can we identify the issues upon

[16] For examples and discussion, see Mayhew (1966) and Ferejohn (1974).

which it takes place? By providing a rigorous definition of logrolling, public choice allows us to answer these questions.

If issues d and s are involved in a logroll, then we know from logrolling's definition that both must pass with the traded votes and fail without them. A supporter of s who trades her vote on d for votes on s votes against her own and/or her constituents' preferences on d. This vote costs her something and she will trade it only if she gets something more valuable in return – enough votes to secure s's victory. It follows that she would not trade her vote on d if s loses even with the trade, and thus we should observe no trading on losing issues. Furthermore, she should not trade her vote on d if s can win without the trade, and thus we should observe no trading on issues that win by substantial margins. The votes on issues involved in logrolling should be close and successful, and the margin of success should be provided by the traded votes.

Stratmann (1992b) has tested these implications of logrolling with data on various votes on the 1985 Farm Bill in the U.S. House of Representatives. It is common practice to explain how a congressman votes by sets of variables that measure the characteristics of the *district* from which he comes, x_D, and characteristics of the *candidate* (for example, ideology), x_C. Thus, in trying to explain voting on three farm bill amendments that would affect peanut farmers (p), dairy farmers (d), and sugar farmers (s) without taking into account the effects of logrolling, one might estimate the following system of equations:

$$\begin{aligned} p &= a_p + b_p x_D + c_p x_C + u_p \\ d &= a_d + b_d x_D + c_d x_C + u_d \\ s &= a_s + b_s x_D + c_s x_C + u_s. \end{aligned} \tag{5.36}$$

If logrolling occurred on these three amendments, however, then the probability that someone who supports farm interests on sugar voting for farm interests on dairies should be higher than that predicted simply by his personal and his district's characteristics. This implication of logrolling can be tested by adding the predicted votes on the other two bills to each equation in (5.36) to obtain (5.37).

$$\begin{aligned} p &= a_p + \beta_p \hat{d} + \gamma_p \hat{s} + b_p x_D + c_p x_C + u_p \\ d &= a_p + \alpha_d \hat{p} + \gamma_d \hat{s} + b_d x_D + c_d x_C + u_d \\ s &= a_s + \alpha_s \hat{p} + \beta_s \hat{d} + b_s x_D + c_s x_C + u_s \end{aligned} \tag{5.37}$$

where $\hat{p}$, $\hat{d}$, and $\hat{s}$ are the predicted votes on each amendment from (5.36).[17] Table 5.4 presents some of Stratmann's results.

As measures of district and congressman's characteristics Stratmann used the amount of campaign contributions each candidate received from the respective farm group's political action committee (PAC), the fraction of the district's population that is engaged in peanut (respectively, dairy and sugar) farming (Farmer), and the

17 Kau and Rubin (1979) suggest adding the actual votes on the other issues, but this approach gives biased estimates of the coefficients on the logrolling variables.

Table 5.4. *Econometric evidence of the presence of logrolling*

Dependent variable	$\hat{p}$	$\hat{d}$	$\hat{s}$	Explanatory variables			
				Const	PAC	Farmer	Party
p		.36*	.53*	−.15	−1.04	71*	−.84*
d	.01		.21*	.14	.18*	.67*	−.72*
s	.45*	.30*		−.33*	1.37*	6.6	.23

Source: Stratmann (1992b, Table 1).

party affiliation of the representative (Republican = 1, Democrat = 0).[18] *Const* is the constant or intercept. An asterisk indicates that the coefficient was significant at the 5 percent level or better. The dependent variable was a one if the representative voted with the farm interests, a zero if he voted against them.

Focusing first on the significant exogenous variables, we see that the probability that a congressman voted in favor of a farm group's interests rises with the amount of contributions that he receives from its PAC (dairies and sugar), and the fraction of his district engaged in this sort of farming (peanuts and dairies). Republicans voted against farmer interests on the peanuts and dairy amendments with a high probability.

Turning to the key logrolling-hypothesis variables, we see that five of the six predicted votes on the other two farm amendments are significant in the three equations. Also, the coefficients are quite large. The probability that someone who was predicted to vote for the sugar amendment also voted for the peanut amendment was 0.53 over and above that predicted on the basis of the candidate and district characteristics included in the model. As one might expect, the congressmen who are predicted to have switched their votes as a result of the trades had estimated probabilities of voting for the respective amendments that fell in the 0.3 to 0.5 range without the trades. These congressmen would, presumably, have to be offered less to switch their vote than would congressmen who were predicted to have only a 0.0 to 0.3 probability of voting for the respective amendments without the trades (Stratmann, 1992b, p. 1171).

Stratmann did not report the $\hat{p}$, $\hat{d}$, and $\hat{s}$ values, but the coefficients in Table 5.4 and the *actual* votes in the three bills can be used to obtain estimates of these variables: $\hat{p} = 61$, $\hat{d} = 207$, and $\hat{s} = 176$. Based only on the votes generated by the district and congressmen's characteristics, farm interests would have lost on both the peanut and sugar amendments, and would have squeaked through on the dairy amendment by a 207 to 205 margin. It is interesting that the only insignificant codetermined variable in the three equations was for $\hat{p}$ in the dairy equation. Votes from supporters of peanut farmers' interests were not needed by the dairy interests, and they do not appear to have demanded them. Votes from the sugar interests converted a narrow 207 to 205 victory into a 245 to 167 victory by our rough calculations, and logrolling spelled the difference between victory and defeat on the other two amendments.

[18] A congressman's ACLU rating was also included to measure his ideology, but it was insignificant in the equations reported here and is ignored.

Stratmann also tests for the presence of logrolling on a dairy amendment that the dairy interests won by a 351 to 36 margin, and on a wheat amendment, on which the farm interests suffered a 251 to 174 defeat. As the theory predicts, no evidence of vote trading is found on these two issues.

"Logrolling" is a distinctly American expression and, as we have just seen, does appear to take place in the U.S. Congress. It does not appear to be unique to that legislative body, however. Elvik (1995), for example, claims that it can explain the distribution of highway expenditures across Norway, a distribution that benefit-cost ratios and the like fail to account for.[19]

5.12 Agenda manipulation

5.12.1 *Agenda control in a spatial environment*

Surely by now the patient reader has grown weary of cycling theorems. Yet we have but scratched the surface of a vast literature establishing cycling and instability results of one form or another. That majority rule leads to cycles has been a (some would say *the*) major theme of the public choice literature. Yet is the problem that serious? Do committees really spin their wheels endlessly as the cycling results seem to suggest? Probably not, and we shall consider several reasons why committees avoid endless cycles in the next section. But before we do let us examine some results that illustrate the potential significance of the cycling phenomenon.

In an important paper, McKelvey (1976) first established that when individual preferences are such as to produce the potential for a cycle with sincere voting under majority rule, then an individual who can control the agenda of pairwise votes can lead the committee to any outcome in the issue space he chooses. The theorem is developed in two parts. First, it is established that with a voting cycle it is possible to move the committee from any starting point S an arbitrarily large distance d from S. In Figure 5.16, let A, B, and C be the ideal points for three voters and S the starting point. If each individual votes sincerely on each issue pair, the committee can be led from S to Z to Z' to Z'' in just three steps. The farther one moves away from S, the larger the voter indifference circles and the larger the steps will become. The process can continue until one is any d one chooses from S.

Now let r be the radius of a circle around S such that (1) the target point of the agenda setter is within the circle (say, ideal point A) and (2) at least $n/2$ ideal points for the committee (in this case two) are within the circle of radius r. Now choose d such that $d > 3r$ and one is certain that a majority of the committee favors A over the last Z^n obtained in the cycle, the $Z^n d$ distance from S. The last pairwise choice offered the committee is then Z^n versus A, and A wins. The agenda setter then either calls a halt to the voting or picks new proposals that will lose to A. Thus, a member of a committee with the power to set the agenda can bring about the victory of his most preferred outcome.

McKelvey's theorem has two important implications. First, and most obviously, the power of the agenda setter may be substantial. If this power is vested in a given

[19] See also Fridstøm and Elvik (1997).

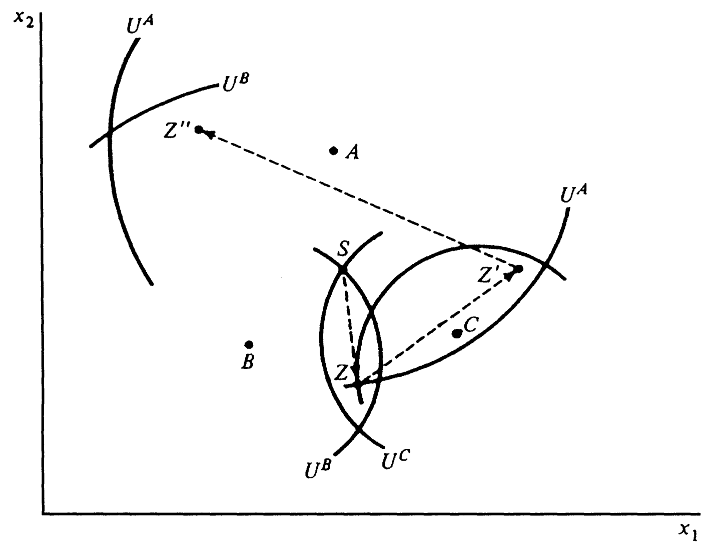

Figure 5.16. Agenda manipulation possibilities.

individual or subcommittee, then one must take precautions lest those with agenda-setting power secure a disproportionate share of the gains from collective action. Second, the existence of a voting cycle introduces a degree of unpredictability to the voting outcomes that may provide an incentive for some to manipulate the process to their advantage. The fact that a committee has reached a decision may in itself not have much normative significance until one learns by what route it got there.

5.12.2 *Agenda control in a divide-the-cake game*

Harrington (1990) has demonstrated the potential power of an agenda setter under quite different conditions than those assumed in Section 5.12.1. Imagine that a committee is offered a gift of G dollars to be divided among its n members. The procedure for selecting a division of G is as follows: one member is selected at random to propose a division of G. If m or more members of the committee, $1 \leq m \leq n$, vote for this proposal it is implemented and the game is over. If the proposal fails to receive at least m votes, another committee member is chosen at random to make a new proposal, and the process continues until some proposal secures the required m votes. To simplify the discussion, let us assume that all members have identical preferences.

Consider first the strategy of a person selected to propose a division of G. She can expect that each member of the committee has some reservation price, that is, some minimum amount x, that he will vote for rather than wait for the outcome of another round. Because all members have identical preferences, whatever one person accepts, all accept. Thus, the proposer maximizes her payoff as the agenda setter by proposing x for $m - 1$ members, $G - (m - 1)x$ for herself, and nothing for the remaining $n - m$ committee members, assuming that her share of G is greater than the common reservation price x.

Now consider the calculus of a member of the committee when deciding what his reservation price should be. He knows that in any round of the game he has a $1/n$ chance of being the proposer and obtaining $G - (m - 1)x$, an $(m - 1)/n$ chance of being any other member of the winning coalition and getting x, and an $(n - m)/n$ chance of getting nothing. If a member were risk neutral and had no preference for present income over future income, he would simply choose a reservation price that equals his expected payoff in any round of the game,

$$x = \frac{1}{n}[G - (m - 1)x] + \frac{m - 1}{n}x + \frac{m - n}{n} \cdot 0. \tag{5.38}$$

Then x would equal G/n, and the proposer's payoff would be

$$\left(\frac{n - m + 1}{n}\right) G. \tag{5.39}$$

The proposer's share of G exceeds that of all other members of the committee so long as $m < n$, and grows as m falls until it reaches one-*half* of the amount to be distributed under the simple majority rule.

If members of the committee are risk averse or have positive time preferences, they will accept some positive x less than G/n in any round rather than run the risk and incur the delay of waiting for another round of the procedure. Thus, the expression in (5.39) constitutes a *lower bound* for the proposer's payoff. The more risk averse and impatient the committee members are, the greater the advantage of the agenda setter.

Harrington is also able to demonstrate the same sort of advantage for an agenda setter under alternative assumptions about how the division game is played. These results are important in that they do not depend on the agenda setter's having an entrenched position due to seniority or the like. Even a randomly selected agenda setter can have a significant advantage over the other committee members. The results do depend crucially on the use of a qualified majority rule that falls short of requiring full unanimity, and thus again illustrate the potential of the unanimity rule to protect the interests of all members of a committee, this time against a selfish agenda setter.[20]

5.13 Why so much stability?

If cycling problems are as pervasive as the public choice literature implies, then why do committee outcomes in Congress and in state legislatures seem to be so stable, both in the sense that the committees do reach decisions and that these outcomes do not gyrate from one meeting of the committee to the next, and from one session of the legislature to the next? This challenging question was put forward by Tullock (1981) and we shall take it up on more than one occasion in this book.

[20] Additional constraints could be imposed, of course. Buchanan and Congleton's (1998) generality principle would require equal treatment of all members, and thus an equal division. Even adding the requirement that a proposal be seconded reduces the agenda setter's power somewhat. See also Baron and Ferejohn (1987).

In Section 5.12 we already encountered one answer to this question, and not a comforting one. An agenda setter may lead the committee to an outcome particularly pleasing to the agenda setter, and keep it there. This solution to the cycling problem is one of several possible answers to Tullock's question that rely on a particular institution like the agenda setter to structure the voting sequence so as to avoid cycles. Robert's Rules of Order and other similar committee procedures are probably the most familiar examples of institutional constraints on a committee that by restricting the possibility of defeated proposals reappearing on the agenda, limit the scope for cycles. We discuss two additional examples of structure-induced equilibria later in this section. But we first consider the simplest of all explanations for the absence of cycles – the nature of the issues themselves precludes them.

5.13.1 *Issues are indeed of one dimension*

As one contemplates the sorts of issues that typically come up in a legislative assembly, the number of potential dimensions of the issue space seems almost unbounded. Defense appropriations involve considerations of national security; a tax on carbon dioxide emissions involves trade-offs between economic growth and environmental protection; a ban on smoking in public places involves considerations of national health and individual liberties. Despite the seemingly boundless range of concerns these sorts of issues raise, an individual's views on these issues often seem to be highly correlated. Once one knows that a representative has voted for a substantial increase in defense spending and against the tax on carbon dioxide emissions, one can predict that this representative will vote against the smoking ban. To the extent this is true, this suggests that the number of dimensions of the "issue" space is much smaller than it first appears. There are but a few underlying "ideological" dimensions, like the familiar liberal–conservative dichotomy, that will allow us to explain and predict how congressmen vote.

Poole and Rosenthal (1985, 1991) have developed a procedure that they call NOMINATE, that allows them to apply factor analysis techniques to data on congressional voting to uncover the underlying "ideological" dimensions of the issue space. They succeeded in correctly classifying some 81 percent of the votes in the U.S. Senate and 83 percent of the votes in the House between 1789 and 1985 with a single dimension.[21]

Poole and Smith (1994) use NOMINATE to identify the most salient dimension of the issue space, and then present evidence that supports both the median voter theorem and the usefulness of concentrating upon a single dimension of the issue space. Their results can be illustrated with the help of Figure 5.17. Suppose a representative's ideal point on a given issue has been identified as lying at point R, where M has been identified as the median position in this dimension and S is the status quo. Then this representative knows that if she proposes her ideal point, it will lose to the status quo. A representative who seeks to make a winning proposal will thus propose a *compromise* like C that is not at her ideal point, and is closer to

[21] See also Hinich and Pollard (1981), Poole and Romer (1985), Laver and Schofield (1990), Enelow and Hinich (1994), and Hinich and Munger (1994).

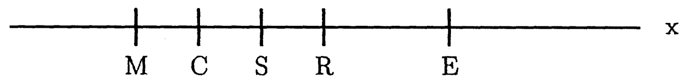

Figure 5.17. Issue proposals along a single-dimensional issue space.

M than *S* is. In contrast, a representative who seeks merely to make an "ideological statement" of her principles proposes her ideal point *R* and suffers defeat. Poole and Smith report evidence consistent with these predictions. Eighty-one percent winning proposals in the Senate were closer to the median position on the issue up for a vote than was the status quo; 62 percent of the losing proposals were farther away. Sponsors who "wanted to win" offered compromises that were closer to the median than their ideal points. Poole and Smith's ability to collapse the diverse issues that come up in the Senate using NOMINATE into a single dimension and accurately predict how senators will vote using this single-dimensional issue space demonstrates the saliency of this one dimension. The fact that senators make proposals and vote in this single-dimensional issue space as the median voter theorem predicts suggests that its equilibrium prediction may hold in the Congress.

Ladha (1994) also employs NOMINATE to identify representative positions, and confirms the predictions of the single-dimensional, medial-voter model. Ladha finds that a series of amendments to a proposal that moves it from *E* to *R* to *C* results in a narrowing of opposition to the amendments with voters on the far right and left not changing their votes, while those toward the center do switch as the amended proposals pass over their identified ideal points.

These results help to establish the predictive content of the medial-voter model, and our trust in the usefulness of assuming that the relevant issue space is single-dimensional. Nevertheless, virtually all studies that have tested for the presence of more than one underlying dimension to the issue space have found more than one.[22] The potential for cycles cannot be dismissed completely on the grounds that all issues involve essentially a division along a single, left–right ideological line.

5.13.2 *Voting one dimension at a time*

The median voter theorem requires both a single-dimensional issue space and single-peaked preferences. If the issue space were known to be, or constrained to be, of one dimension – expenditures on space exploration – the single-peaked preferences assumption would not seem to be a major concern. What is implausible is the assumption that the issue space is one-dimensional.

Suppose, therefore, that we have a two-dimensional issue space, but that we limit voting to but one dimension at a time. Consider Figure 5.18, where x_1 and x_2 are two public goods vectors. Tax rates to finance the public goods are assumed given, so that *A*, *B*, and *C* are again the ideal points of our three voters. With each voter free to propose any point in the positive orthant, a cycle can ensue. But let the committee

[22] See, again, Poole and Rosenthal (1985, 1991), Poole and Romer (1985), Laver and Schofield (1990), and Hinich and Munger (1994); and for a direct critique of the NOMINATE procedure regarding its implied underlying dimensionality, Koford (1989, 1990).

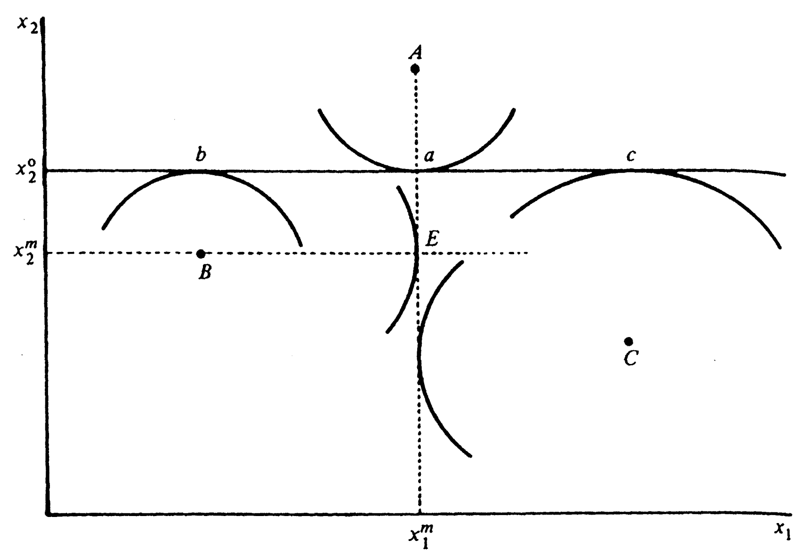

Figure 5.18. Equilibrium outcomes with sequential votes.

rule be that voting must take place one dimension at a time. Take x_2^o as initially given and have the committee vote on the level of x_1, given x_2^o. With circular (or ellipsoid) indifference contours, each voter has single-peaked preferences along the horizontal line x_2^o. B favors point b, A favors a, and C favors c. A is the median voter in the x_1 dimension and x_1^m is the quantity of x_1 chosen under majority rule. Now fix x_1 at x_1^m and allow the committee to decide the quantity of x_2. B is now the median voter and x_2^m is the quantity of x_2 chosen. Point E is an equilibrium under majority rule given the constraints that x_1 and x_2 must be voted upon one dimension at a time.[23]

With tax shares fixed, the Pareto set is the triangle with apexes at A, B, and C. E falls inside this triangle and is Pareto optimal under the constraint that tax shares are fixed. But taxes are one of the important variables a committee must decide. If the choice of tax rate can be formulated as a one-dimensional issue – say, the degree of progressivity of an income tax – then tax progressivity can be voted on as a separate issue, holding x_1 and x_2 constant, and an equilibrium outcome chosen in these three dimensions. But this equilibrium outcome need not be Pareto optimal (Slutsky, 1977b). To find the Pareto-optimal quantities of x_1 and x_2, one chooses x_1, x_2 and the individual tax shares so as to maximize the sum of the utilities of the committee. The resulting solution must satisfy the Samuelsonian condition for the Pareto-optimal allocation of a public good. Choosing quantities of each public good and tax rates one dimension at a time in effect adds further constraints to the maximization problem. There is no reason to suspect that this constrained committee choice will coincide with what the unconstrained solution would be, and in general it will not. The price of an equilibrium under majority rule can be high.

[23] E is a median in two directions. To be an unconstrained equilibrium, it must be a median in all directions, which it is not. Allow the committee to vote on combinations of x_1 and x_2 along a ray through E running in a northeast direction, and E will not be the chosen point.

5.13.3 *Logrolling equilibria*

The theorems that logrolling situations imply voting cycles and that agenda setters can achieve their ideal points in cyclic situations assume that every voter at each step of a voting sequence votes sincerely. Voters, like the children of Hamlin, follow the agenda setter blindly wherever he goes. These theorems assume a seemingly unrealistic degree of myopia on the part of voters.

Consider again the trading cycle illustrated with the help of Tables 5.2 and 5.3. B first agrees to trade votes with C, then deserts her for A, who in turn jilts B for C. For a true cycle to enfold, B and C, not having learned their lessons, must again agree to swap votes and we repeat the cycle. But surely rational individuals should not allow themselves to be dragged through too many revolutions of this cycle before they begin to foresee the short-run nature of each trade. Once each trader realizes that an apparently advantageous trade is likely to be overturned, he might try to stick to a *relatively* advantageous pair of trades once made, or never allow himself to be talked into a trade to begin with. Note, in this regard, the inherent instability of the outcome pairs $(X, \sim Y)$ and $(\sim X, Y)$. Under each of these two outcomes one individual (B or C) gets her maximum potential gain. Thus, were the coalition A–B to form to produce $(X, \sim Y)$ as the outcome pair, A can threaten to leave B, since both A and C are better off when they form a coalition than when A stays with B. But B's only alternative to A is a coalition with C, which makes B worse off. Thus, B prefers preserving the A–B coalition, but if A is rational, B will be incapable of doing so. Now consider the B–C coalition to produce (X, Y). Either B or C could become better off by joining A to produce $(X, \sim Y)$ or $(\sim X, Y)$, respectively. Both have the identical threats to make against the B–C coalition. Thus, if one individual begins to waver in her support for the B–C coalition, the other can issue the counterthreat to bolt and joint with A. Since both are confronted by the same threats and counterthreats, each may decide that it is better to remain in the B–C coalition.

Considerations such as these lead one to predict a coalition between B and C with outcomes (X, Y), even though no core exists. This outcome is contained in the main solution concepts, which have been proposed to solve simple bargaining games (e.g., the von Neumann-Morgenstern solution, the bargaining set, the kernel, and the competitive solution). If vote trading in parliamentary committees resembles the kinds of bargaining deliberations that underlie these different solution concepts, then stable, predictable outcomes from a logrolling process can be expected even though no core exists and myopic trading would produce a cycle. Oppenheimer (1979) has argued in favor of the bargaining set as predictor of outcomes from logrolling, whereas McKelvey and Ordeshook (1980) have found that outcomes from vote-trading experiments conform to the competitive solution.

In the game depicted in Tables 5.2 and 5.3, either voter B or C could ensure the outcome $(\sim X, \sim Y)$ that arises when each voter sincerely states her true preferences by voting against both issues. If B, say, votes against X and Y, A can achieve her most preferred outcome $(\sim X, \sim Y)$ by voting sincerely. C can make her no better

Matrix 5.1. *Logrolling options*

		Voter C	
		Vote for X and Y	*Vote for Y and against X*
Voter B	Vote for X and Y	1 (+3, +3)	2 (−2, +5)
	Vote for X and against Y	3 (+5, −2)	4 (0, 0)

proposal and ($\sim X$, $\sim Y$) will be the committee choice. Thus, if B or C were fearful that trading would produce an outcome that left them worse off than the sincere voting outcome ($\sim X$, $\sim Y$), they could make sure that this outcome comes about by following the *sophisticated* strategy of voting against both issues.[24] Enelow and Koehler (1979) show that the majority, which produces the sincere voting outcome, always can preserve this outcome by the appropriate sophisticated voting strategy, even when logrolling with sincere voting would overturn it.

Thus, there is reason to suspect that either (X, Y) or ($\sim X$, $\sim Y$) would emerge as the committee outcome in the example from Tables 5.2 and 5.3. Although either B or C can preserve ($\sim X$, $\sim Y$) by sophisticated voting, the temptation to join with one another to produce (X, Y) must be strong. What might prevent them from ever doing so is the fear that once the B–C coalition has formed, the other trading partner will fail to deliver on her part of the trade (or join with A). This danger is particularly likely when issues X and Y are decided sequentially. We have here another example of a prisoners' dilemma (Bernholz, 1977). Matrix 5.1 depicts the strategic options for voters B and C when issues X and Y must be decided as before. Both voters are better off with the trade (square 1) than without it (square 4), but the incentive to cheat is present. If issue X is decided before issue Y and voter C lives up to her part of the bargain by voting for X, the outcomes in column 2 become infeasible. Voter B must choose between squares 1 and 3, and her choice is obvious if there is no possibility for voter C to retaliate.

As we have seen in Chapter 2, the cooperative solution to the prisoners' dilemma emerges only if each player thinks that her choice of the cooperative strategy is likely to induce the corresponding strategy choice of the other player. If the strategy options are played in sequence and the game is played but once, the first player has no means by which to influence the second player's decision at the time the latter is made. Thus, one would not expect vote trading to take place over issues decided sequentially among coalitions that form but a single time. A stable, cooperative

[24] The distinction between sincere and sophisticated voting was introduced by Farquharson (1969). In a sequence of pairwise votes, an individual votes *sincerely* if at each step in the sequence she votes for the element of the issue pair that she prefers. An individual votes *sophisticatedly* at each step if she determines the optimal strategy by considering all future steps in the sequence and the future behavior of the other players. Sophisticated voting requires the individual to engage in backward induction and to eliminate all weakly dominated strategies from consideration.

vote-trading game can be expected only when the issues on which votes are traded are all decided simultaneously, say, as part of an omnibus highway bill; or when the same constellations of issues come up time and time again, and a prisoners' dilemma supergame emerges. Bernholz (1978) has discussed the latter possibility. Under the assumptions that the same types of issues do arise again and again, he shows that the likelihood of a stable prisoners' dilemma supergame emerging is positively related to both the net potential gains from cooperation and the probability that the same players reappear in each successive game. As Bernholz notes, the depiction of logrolling situations as single plays of a prisoners' dilemma supergame is plausible for a legislative assembly, whose members continually represent the same interest and have reasonably long tenure.

In Section 5.11 we discussed evidence indicating that vote trading had in fact taken place across three amendments to a farm bill despite our proof in Section 5.10 that the existence of these very trades demonstrates the presence of an underlying set of preferences that would produce a cycle under the majority rule. What or who prevented the cycle from destroying the set of trades that transpired? The "what" might be the procedures through which bills are brought to a floor vote. The "who" is almost certainly the two parties' leadership. Arranging vote trades and ensuring that bargains are kept is the job of party leaders and their whips. These "agenda setters" are elected to their posts by their fellow party members presumably in part on the basis of how capable they are at avoiding cycles and satisfying the goals of all party members, not just the leaders. Both Haefele (1971) and Koford (1982) see party leadership as effectively guiding the legislature to outcomes that maximize the welfare of the party membership. Their rather optimistic description of how the legislative process functions stands in sharp contrast to most of the logrolling-majority rule-cycling literature.[25]

5.13.4 *Empirical evidence of cycling*

We have reviewed theorems that imply that cycling is almost inevitable, and arguments why it might not occur at all. Which are correct? Is cycling truly rare, as Tullock's rhetorical question (why so much stability?) assumes, or can it in fact be observed? We close this chapter by examining two sets of evidence pertaining to the presence of cycles. In this subsection we look at some evidence from the U.S. Congress; in the next we look at evidence from the experimental laboratory.

A cycle exists when y defeats x, z defeats y, and x in turn defeats z. Few committees are likely to be so dense as to propose precisely the same x that was defeated in an earlier vote against y. Cycling is more likely to manifest itself by a proposal that comes close to x defeating z, which then loses to a proposal that resembles y. Detecting cycles by examining the content of individual proposals is likely to be a long and tedious task.

[25] The same can be said of the model of vote trading recently developed by Philipson and Snyder (1996), who assume the existence of an auctioneer/party leader who arranges trades between high- and low-intensity voters on a single-dimensional issue to achieve an equilibrium at which the summed utilities of the voters are maximized. Mueller, Philpotts, and Vanek (1972) established a similar result by simulating Walrasian vote markets.

Table 5.5. *Predicted payoffs and variances of payoffs with and without cycling*

Issues	Voter 1	Voter 2	Voter 3	Variance
		A. With Cycling		
1	0.75	0.25	0	0.097
2	0	0.75	0.25	0.097
3	0.25	0	0.75	0.097
Sum	1	1	1	0
	Sum of individual variances (3 × (0.097)) = 0.292			
		B. With Stable Coalition		
1	0.5	0.5	0	0.055
2	0.5	0.5	0	0.055
3	0.5	0.5	0	0.055
Sum	1.5	1.5	0	0.5
	Sum of individual variances (3 × (0.055)) = 0.167			

A cycle can leave evidence of itself of another form, however. The identities of members of the winning coalition should change over time, as well as the distribution of the payoffs to the committee. Consider again the simple three-person-divide-the-dollar game discussed earlier. Part A of Table 5.5 presents payoffs that we might expect to see in the presence of a majority rule cycle. Players 1 and 3 form a winning coalition on the first issue, 1 and 2 on the second, and so on. The outcome of a vote on any single issue involves a quite asymmetric distribution of the dollar, with one player receiving at least one-half and another nothing. Thus, the variance in the payoffs from the vote on any issue could be large and the sum of the variances should grow over time. When a cycle is present, however, a player who loses in one round of voting should win in a subsequent round, and thus the aggregate payoffs in the presence of a cycle should be much more evenly divided than the payoffs in any round, and the variance of the sum of payoffs should be much less than the sum of the variances from the individual rounds.

Part B of Table 5.5 presents the pattern of payoffs one might expect in the absence of a cycle, when a stable coalition exists. We again expect an uneven distribution of the dollar in any single round, and thus a positive variance in the payoffs in this round, but now we expect the same distribution of payoffs to persist over time. The variance of the sum of the payoffs will, therefore, not be less than the sum of the variances from the individual rounds as predicted when a cycle exists, but much greater.

Stratmann (1996a) has used these implications of cycling to test for its presence in the pattern of federal grants to congressional districts in the United States between 1985 and 1990. These federal programs contain the major categories of pork-barrel legislation, and thus can be regarded as largely redistributive in nature, and thus likely to exhibit cycling if it exists in the Congress. Table 5.6 presents some of his findings. The first thing to note is that the payoffs to congressional districts are quite unevenly distributed. In every year a *minority* of districts benefits from a given grant

Table 5.6. *Characteristics of federal grants to congressional districts, 1985–90*

Year 1	Number of programs 2	Programs benefiting a minority of districts 3	Benefiting programs benefiting a minority of districts (%) 4	Variance of sums 5	Sum of variances 6
1985	592	543	91.7	5.6 E16	7.1 E15
1986	624	571	91.5	2.7 E16	5.5 E15
1987	637	575	90.3	2.2 E16	5.6 E15
1988	679	616	90.7	2.4 E16	6.4 E15
1989	706	646	91.5	2.7 E16	6.6 E15
1990	791	724	91.5	4.1 E16	7.2 E15

Source: Stratmann (1996a, Tables 5 and 6).

program for more than 90 percent of the programs. In 1989 the mean federal grant to the ten districts that benefited the most from these programs was $968 million – *over 75 times more than* the 10 districts benefiting least from the program averaged. Columns 5 and 6 in Table 5.6 reveal that the variance of the sums of the payoffs in any year are from four to nine times the sum of the variances in that year, contradicting a prediction that cycling occurred across these grant programs in any given year. The correlations across payoffs over time are 0.9 or better, suggesting that cycling did not occur over time (Stratmann, 1996a, p. 25).[26] Stratmann's findings strongly imply that a stable coalition existed in the U.S. Congress between 1985 and 1990 when it came to the disbursement of federal grants.

Although these results suggest a "tyranny of the majority" in the U.S. Congress, they still present some puzzles. Why, for example, would a district whose representative was left out of the winning coalition receive any grants at all? Why are so many of the votes on these pork-barrel programs so lopsided?[27] Several authors have answered these questions by arguing that a norm of *universalism* exists in the Congress.[28] Rather than encourage cycles and run the risks of losing out by forming majority coalitions that pass redistributive legislation by narrow majorities, a coalition of the whole forms and everyone is allowed a share of the funds that flow from Washington.

Although "universalism" is an appealing way out of the paradox of near-unanimous support for redistributive programs, it too is not without its problems. One does not usually expect universal norms to dictate that one person's share *ought* to be 75 times greater than that of the next person. Indeed, when one factors in the taxes each district pays to finance these programs, many districts – quite possibly a majority – are *net losers*. Why are congressional norms both universal and so unegalitarian?

[26] Van Deemen and Vergunst (1998) also fail to find evidence of cyclic preferences in the data on Dutch national elections for the years 1982, 1986, 1989, and 1994. Kurrild-Klitgaard (2001) did detect the potential for a cycle *if* Danish voters had been allowed to choose their prime minister directly in the 1994 election.

[27] See Ferejohn (1974) and Mayhew (1974, pp. 88–113).

[28] See Weingast (1979); Weingast, Shepsle, and Johnsen (1981); Shepsle and Weingast (1981); and Niou and Ordeshook (1985). This explanation was also part of Tullock's (1981) answer to the stability question.

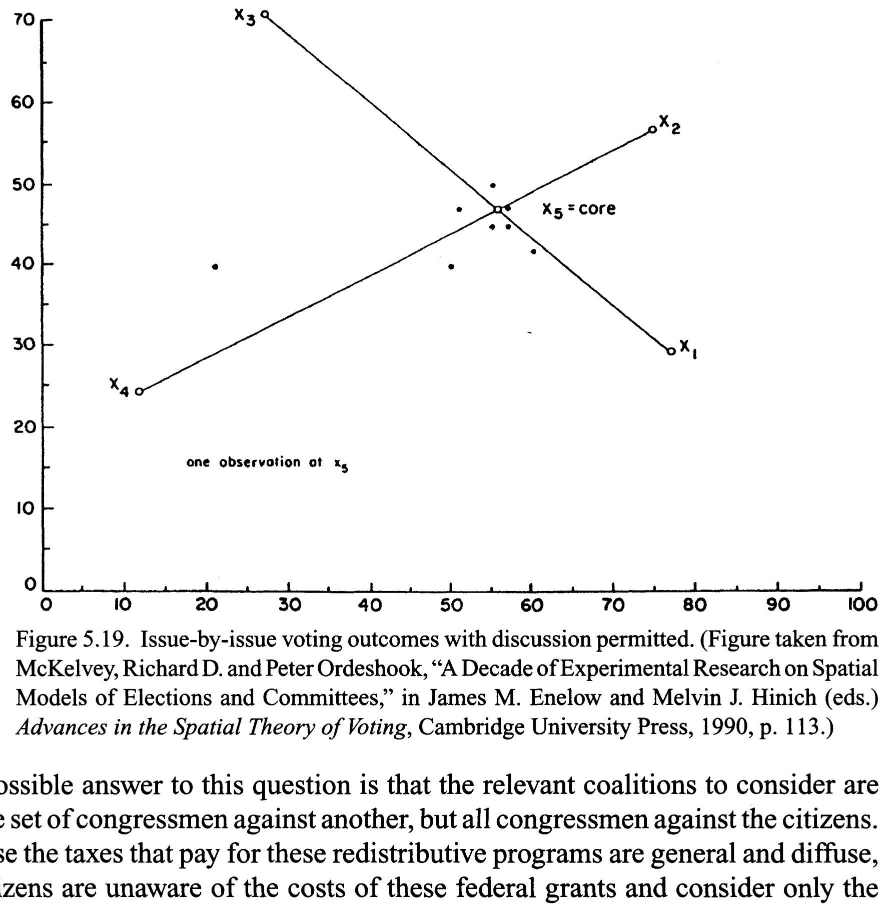

Figure 5.19. Issue-by-issue voting outcomes with discussion permitted. (Figure taken from McKelvey, Richard D. and Peter Ordeshook, "A Decade of Experimental Research on Spatial Models of Elections and Committees," in James M. Enelow and Melvin J. Hinich (eds.) *Advances in the Spatial Theory of Voting*, Cambridge University Press, 1990, p. 113.)

A possible answer to this question is that the relevant coalitions to consider are not one set of congressmen against another, but all congressmen against the citizens. Because the taxes that pay for these redistributive programs are general and diffuse, the citizens are unaware of the costs of these federal grants and consider only the concentrated benefits that they receive. Each congressman is evaluated on the basis of his marginal contribution to the district's welfare and any grants it receives are counted as part of these marginal contributions. Although a congressman whose district receives only $10 million in grants has not won as much as the congressman whose district got $750 million, he has still "won" something. It is only the taxpayer-citizen who loses under this interpretation.[29]

5.13.5 *Experimental evidence of cycling*

The most controlled environment to test for the presence of cycling is within the experimental laboratory, and a variety of experiments have been conducted which bear on this question. Many of these have defined the issue set spatially, as we have throughout much of this chapter. A set of preferences is induced in these experiments by giving participant i a reward of D dollars if the committee chooses a particular point x_i in the two-dimensional issue space, with successively lower payoffs awarded to i the farther the committee's choice is from x_i. Although most studies have induced circular indifference curves, some have induced ellipses and even more exotic shapes.

[29] For a formal modeling of this way around the "universalism paradox" see Schwartz (1994).

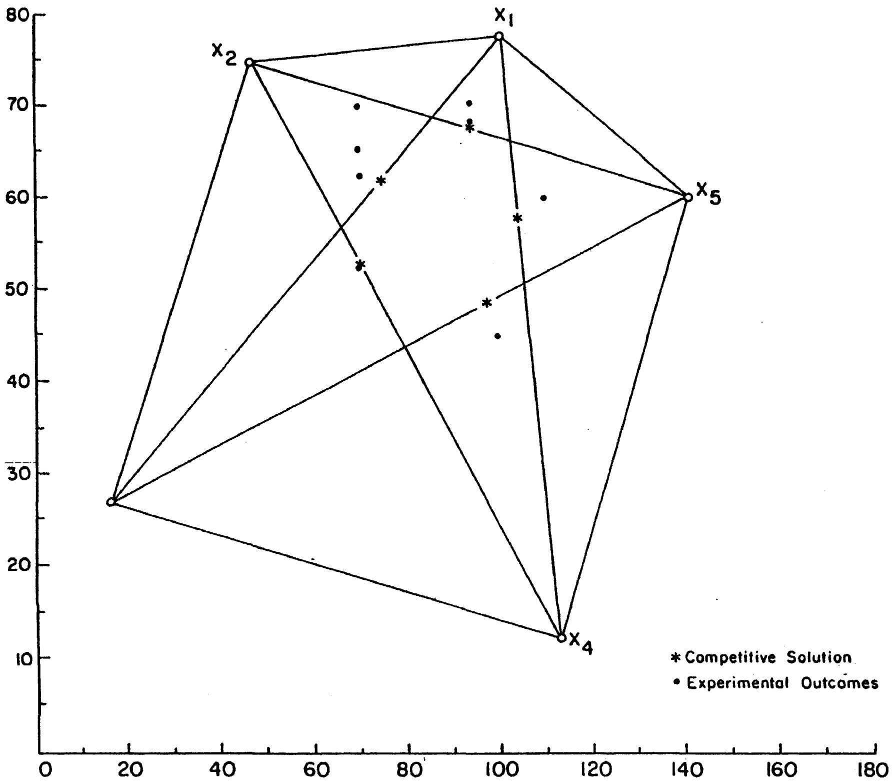

Figure 5.20. Competitive solution test. (Figure taken from McKelvey, Richard D. and Peter Ordeshook, "A Decade of Experimental Research on Spatial Models of Elections and Committees," in James M. Enelow and Melvin J. Hinich (eds.) *Advances in the Spatial Theory of Voting*, Cambridge University Press, 1990, p. 113.)

The earliest experimental results for committee voting test to see whether the committee chooses a Condorcet winner, when one exists. By Plott's (1967) theorem, an equilibrium will only exist in a spatial voting game if there is an odd number of players, and pairs of players are perfectly lined up on opposite sides of one player's ideal point, as in Figure 5.9 where each letter is a voter's ideal point and the unique winning point is at E. Fiorina and Plott (1978) were the first to run experiments of this type and they found that the committee's choices did tend to cluster around this equilibrium (core) outcome, even though they seldom coincided with it. Many subsequent experiments have confirmed Fiorina and Plott's findings. One such set of outcomes by McKelvey and Ordeshook (1987) is presented in Figure 5.19. Each point is an experiment's outcome. The core is at player 5's ideal point, x_5, and most of the points chosen in the experiments are clustered around this point with one falling precisely on top of it. Note, however, that one committee managed to wander off quite a ways to the left.

Thus, it appears that committees do gravitate toward a Condorcet winner when one exists. Where do they locate when one does not exist? One answer, which underlies McKelvey's (1976) agenda setter theorem, is that the committee might wind up anywhere on the page, or several miles from it. But such predictions strain one's credibility. More reasonable would be a prediction that the committee chooses

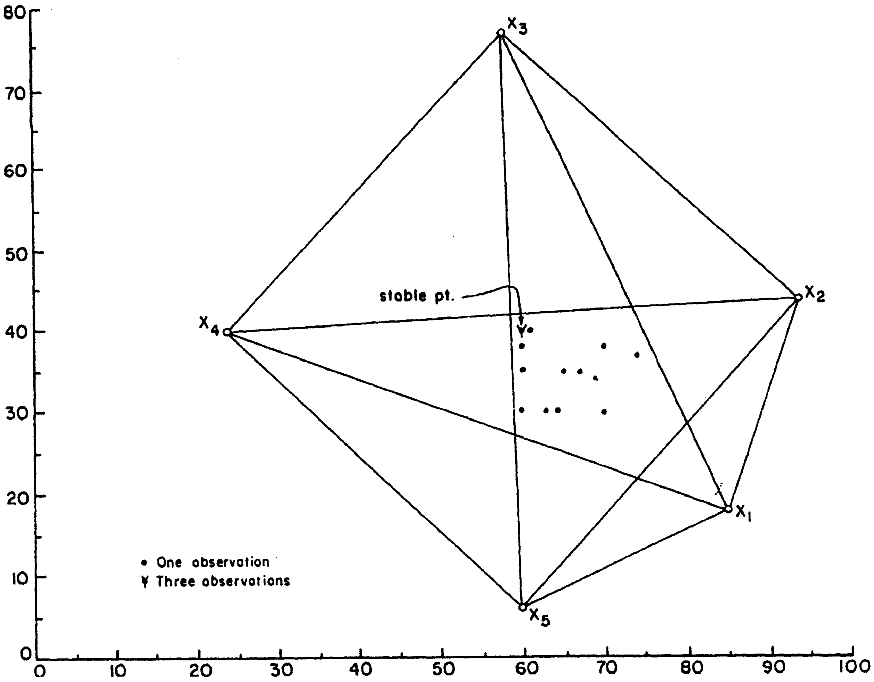

Figure 5.21. Discussion prohibited, issue-by-issue voting. (Figure taken from McKelvey, Richard D. and Peter Ordeshook, "A Decade of Experimental Research on Spatial Models of Elections and Committees," in James M. Enelow and Melvin J. Hinich (eds.) *Advances in the Spatial Theory of Voting*, Cambridge University Press, 1990, p. 113.)

a point somewhere inside the Pareto set, or even somewhere in the middle of this set. Game theory has generated several solution concepts – like the bargaining, uncovered and Banks sets – to predict where this outcome might be. (We shall discuss some of these concepts in Chapter 11, where the issue of cycling will again arise in the context of two-candidate competition.) Figure 5.20 presents the outcomes from a set of experiments by McKelvey, Ordeshook, and Winer (1978) designed to test the predictive power of one of these solution concepts – the competitive solution. All experiments resulted in a point being chosen within the Pareto set, which is the area contained within the large pentagon formed by the outside lines connecting the five ideal points. Each asterisk represents an outcome predicted by the competitive solution. All of the committees' chosen points come close to the predicted outcomes, with a few falling right on them. The McKelvey, Ordeshook, and Winer experiments and many others conducted to test different hypotheses about committee choices when no core exists reveal that the majority rule selects outcomes that are both within the Pareto set, and which tend to cluster near one another, although not as closely together as when a core exists.[30]

In Section 5.13.2 we illustrated how a majority rule equilibrium can be induced with a multidimensional issues space if the issues are voted on one dimension at a time. Figure 5.21 presents the results from yet another set of experiments by

[30] These and other results from the experimental literature on spatial voting are surveyed by McKelvey and Ordeshook (1990).

McKelvey and Ordeshook (1984) that tests this prediction. The "stable point" is at the intersection of the two horizontal and vertical lines that pass through the median ideal points in the two directions. The points chosen in the experiments do not cluster as closely to this stable point as they did to the core point in Figure 5.19, but they are more closely clustered than in Figure 5.20, even though no core exists in each of these experiments, *if* the committees were free to offer new proposals in any way they wished. By constraining the committees to change only one dimension of the proposals at a time, this last set of experiments produced a concentrated cluster of outcomes. Indeed, all of the points chosen fall within the pentagon formed by the intersection of the diagonal contract curves. Even adding a modest amount of structure to a committee's procedures can make a noticeable difference in the stability of its outcomes.

Bibliographical notes

The literature on majority rule is reviewed by Enelow (1997) and Young (1997). A rigorous proof of the median voter theorem is presented by Kramer (1972). Kramer and Klevorick (1974) established a similar result for local optima, and Kats and Nitzan (1976) have shown that a local equilibrium is likely to be a global equilibrium under fairly mild conditions.

Following Plott (1967), the major papers on stability conditions in multidimensional models have been Kadane (1972), Sloss (1973), Slutsky (1977a), Schofield (1978), and Cohen (1979).

Hoyer and Mayer (1974) prove the median-in-all-directions theorem using elliptical indifference curves.

The literature on axiomatic restrictions on preference orderings to produce majority rule equilibria is reviewed in Inada (1969), Sen (1970a), Plott (1971), Taylor (1971), and Pattanaik (1997).

In addition to the papers cited, Simpson (1969) and Kramer (1977) have made important contributions specifying the conditions under which a particular majority, m^*, suffices to ensure an equilibrium.

The seminal discussions of logrolling in public choice are by Downs (1957), Tullock (1959), and most extensively by Buchanan and Tullock (1962). In political science, the classic reference is Bentley (1907).

Arguments that logrolling can, in the proper structural setting, improve on the outcomes from simple, sincere majority voting have been presented by Coleman (1966a,b, 1970); Mueller (1967, 1971, 1973); Wilson (1969, 1971a,b); Mueller, Philpotts, and Vanek (1972); Koford (1982); and Philipson and Snyder (1996).

The negative side of logrolling has figured more prominently in the political science literature (Schattschneider, 1935; McConnell, 1966; Lowi, 1969; Riker and Brams, 1973; Schwartz, 1975).

The theorem relating logrolling to cycling appears in various forms in Park (1967), Kadane (1972), Oppenheimer (1972, 1975), Bernholz (1973, 1974a, 1975), Riker and Brams (1973), Koehler (1975), and Schwartz (1981). A very useful review of this literature with additional proofs is given by Miller (1977). A less technical review of the logrolling literature is given by Stratmann (1997).

Shepsle and Weingast (1981) discuss several possible institutions to bring about "structure-induced" equilibria. Niemi (1983) emphasizes the potential importance of limiting the issue set to a few choices and presents a weakened version of the single-peakedness condition.

Bernholz (1974b) was the first to propose limiting consideration to a single dimension at a time to induce equilibria. Both Slutsky (1977b) and Shepsle (1979) offer proofs of the result.

Coleman (1983) emphasizes the importance of the long-run context in which logrolling takes place in bringing about stability, and Bernholz proves some relevant theorems (1977, 1978, 1997).

The experimental literature on spatial voting is surveyed by McKelvey and Ordeshook (1990).