CHAPTER 12

Two-party competition – probabilistic voting

It suffices for us, if the moral and physical condition of our own citizens qualifies them to select the able and good for the direction of their government, with a recurrence of elections at such short periods as will enable them to displace an unfaithful servant, before the mischief he mediates may be irremediable.

Thomas Jefferson

The social meaning or function of parliamentary activity is no doubt to turn out legislation and, in part, administrative measures. But in order to understand how democratic politics serve this social end, we must start from the competitive struggle for power and office and realize that the social function is fulfilled, as it were, incidentally – in the same sense as production is incidental to the making of profits.

Joseph Schumpeter

The cycling problem has haunted the public choice literature since its inception. Cycling introduces a degree of indeterminacy and inconsistency into the political process that hampers the observer's ability to predict outcomes, and clouds the normative properties of the outcomes achieved. The median voter theorem offers a way out of this morass of indeterminateness, a way out that numerous empirically minded researchers have seized. But the median voter equilibrium remains an "artifact" of the assumption that issue spaces have a single dimension (Hinich, 1977). If candidates can compete along two or more dimensions, the equilibrium disappears and with it the predictive power of the econometric models that rely on this equilibrium concept.

Not surprisingly, numerous efforts to avoid these dire implications of assuming multidimensional issue spaces have been made. Some of these were discussed in the previous chapter. Here we focus upon one set of models that makes a particularly plausible and powerful modification to the standard two-party spatial competition model and produces equilibrium outcomes. We begin by reexamining why the standard model fails to achieve an equilibrium.

12.1 Instability with deterministic voting

Consider again a situation in which there are three voters with ideal points at A, B, and C in the two-dimensional issue space, $x - y$ (Figure 12.1). With separable utility functions, voter indifference contours are concentric circles and the Pareto

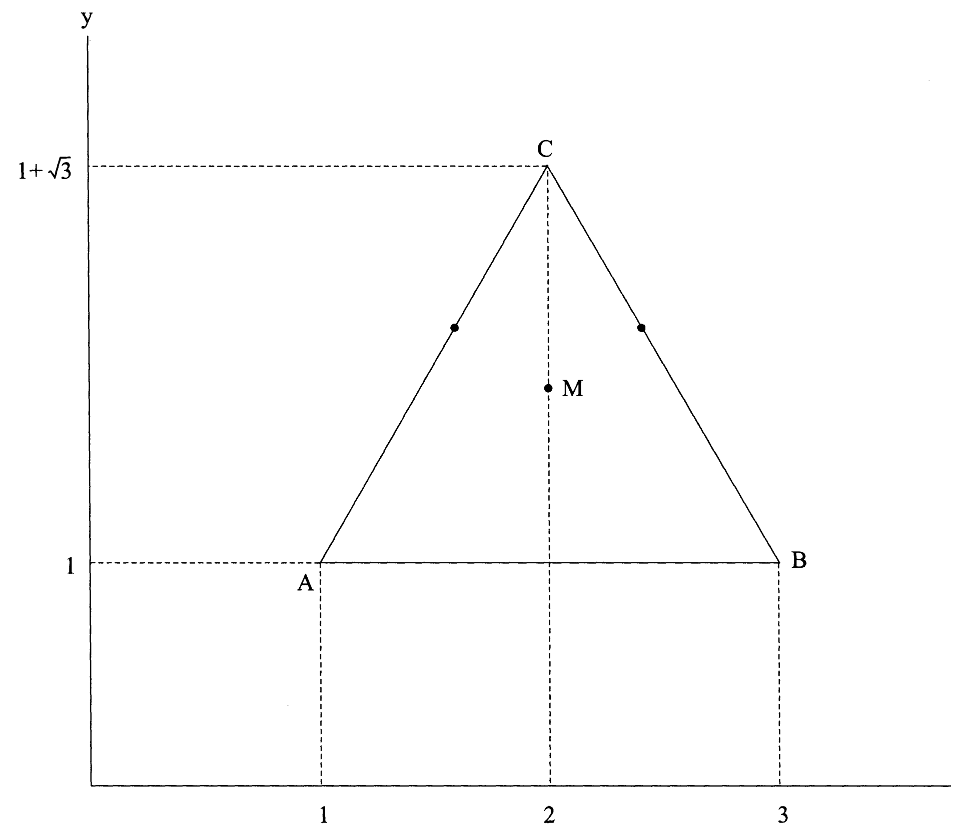

Figure 12.1. Ideal points of three voters.

set is the triangle with apexes at A, B, and C. The two candidates compete by choosing points in the $x - y$ positive quadrant.

Our intuition suggests that the candidates choose points inside ABC. Could a point outside the triangle win more votes than a point inside the triangle, given that the former must always provide lower utility to *all three* voters than some points inside the triangle? Intuition further suggests that competition between the candidates for the three votes drives the two candidates toward the middle of the triangle, to some point like M.

But we have seen in Chapter 5 that point M cannot be an equilibrium if candidates seek to maximize their votes and voters vote for the candidate who takes the closest position to a voter's ideal point. If candidate 1 is at M, then 2 can defeat 1 by taking any position within the three lenses formed by U_A and U_B, U_A and U_C, and U_B and U_C (see Figure 12.2). Note that these lenses include points like N outside the Pareto set. But any point that 2 chooses can be defeated by a countermove by 1, and so on, ad infinitum.

Let us consider again the assumption that each voter votes with certainty for the candidate whose platform is closest to the voter's ideal point. Candidate 1 has taken a position at P_1 in Figure 12.3, and candidate 2 is considering taking positions along the ray AZ. In deciding what point along AZ to choose, 2 contemplates the effect of

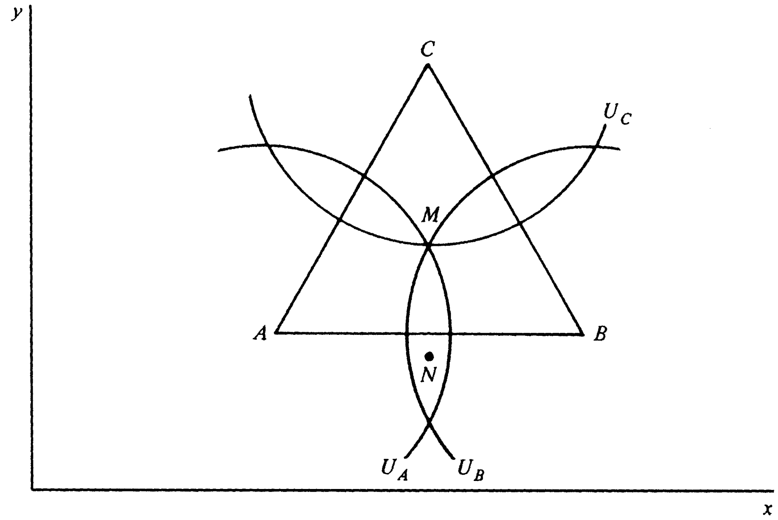

Figure 12.2. Cycling possibilities.

this choice on the probability of winning A's vote. Under the deterministic voting assumption that voter A votes for the candidate closest to point A, this probability remains zero as long as 2 remains outside U_A, and then jumps to one as 2 crosses the U_A contour. The probability of A's voting for 2 is a discontinuous step function equaling zero for all points outside U_A and one for all points inside.

That a candidate expects voters to respond to changes in her platform in such a jerky manner seems implausible for a variety of reasons. First of all, A is unlikely to be perfectly informed about the two candidates' positions, and thus A may not realize that 2 has moved closer to his ideal point. Second, other random events may impinge upon A's decision, which either change his preferences or change his vote

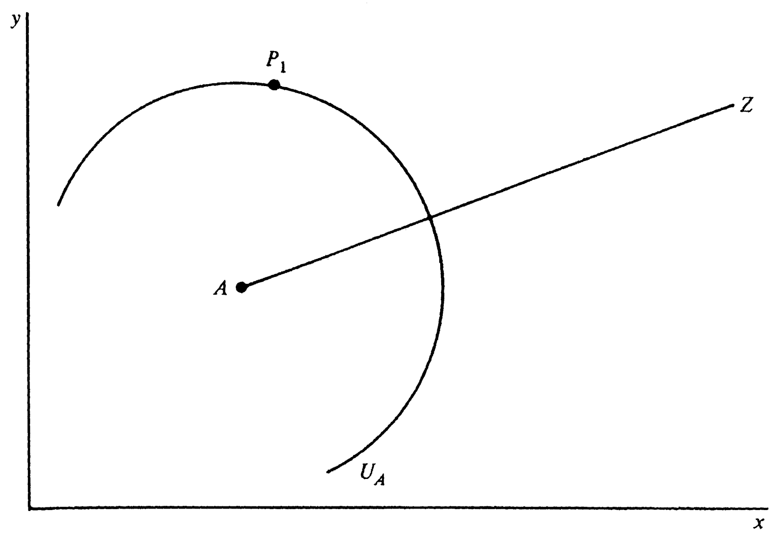

Figure 12.3. Voter A's response to candidate 2's moves.

in an unpredictable way. Third, 2 may not know with certainty where A's ideal point lies. Thus, a more realistic assumption about 2's expectation of the probability of winning A's vote is that it is a continuous function of the distance 2's position lies from A, increasing as 2 moves closer to A.[1]

With this plausible alternative to the deterministic voting assumption, two-party competition for votes can produce equilibrium outcomes.

12.2 Equilibria under probabilistic voting

Deterministic voting models assume that voter choices gyrate schizophrenically as candidates move about competing for votes. A slight movement to the left loses A's vote, but wins B's and C's. Candidates seek to maximize their expected number of votes, and these in turn are simply the sum of the probabilities that each voter will vote for the candidate. Define π_{1i} as the probability that voter i votes for candidate 1, and EV_1 1's expected vote. Then candidate 1 seeks to maximize

$$EV_1 = \sum_{i=1}^{n} \pi_{1i}. \tag{12.1}$$

Under deterministic voting, π_{1i} and π_{2i} take the following step-function form:

$$\begin{aligned} (\pi_{1i} = 1) &\leftrightarrow U_{1i} > U_{2i} \\ (\pi_{1i} = 0) &\leftrightarrow U_{1i} \leq U_{2i} \\ (\pi_{2i} = 1) &\leftrightarrow U_{1i} < U_{2i}, \end{aligned} \tag{12.2}$$

where U_{1i} and U_{2i} are i's expected utilities under the platforms of 1 and 2, respectively.

Probabilistic voting models replace (12.2) with the assumption that the probability functions are continuous in U_{1i} and U_{2i}; that is,

$$\pi_{1i} = f_i(U_{1i}, U_{2i}), \qquad \frac{\partial f_i}{\partial U_{1i}} > 0, \qquad \frac{\partial f_i}{\partial U_{2i}} < 0. \tag{12.3}$$

The task of finding a maximum for (12.1) will be much easier if the π_{1i} are smooth, continuous concave functions, rather than discontinuous functions. The probabilistic voting assumption makes this substitution, and it lies at the heart of the difference between the characteristics of the two models.

The utility functions of each voter can be thought of as mountains with peaks at each voter's ideal point. The probabilistic voting assumption transforms these utility mountains into probability mountains, with the probability of any voter voting for a given candidate reaching a peak when the candidate takes a position at the voter's ideal point.

Equation (12.1) aggregates these individual probability mountains into a single aggregate probability mountain. The competition for votes between candidates drives them to the peak of this mountain.

[1] For further justification of the probabilistic voting assumption, see Hinich (1977); Coughlin, Mueller, and Murrell (1990); and Hinich and Munger (1994, pp. 166–76).

That the positioning of the candidates at the peak of this mountain is an equilibrium can be established in a variety of ways. For example, the zero-sum nature of competition for votes, combined with the continuity assumptions on the π_{1i} and π_{2i} (implying the continuity of EV_1 and EV_2), can be relied upon to establish a Nash equilibrium if the issue space over which the candidates compete is compact and convex (Coughlin and Nitzan, 1981a). If the probability functions are strictly concave, the equilibrium is unique, with both candidates offering the same platforms.

12.3 Normative characteristics of the equilibria

Let us examine the properties of the equilibria further by making some specific assumptions about the probability functions. First of all, we assume that all voters vote so that the probability that i votes for candidate 2 is one minus the probability that i votes for 1; that is,

$$\pi_{2i} = 1 - \pi_{1i}. \tag{12.4}$$

In addition to satisfying (12.3), the probability functions must be chosen so that

$$0 \leq f(\,) \leq 1 \tag{12.5}$$

for all feasible arguments. As a first illustration, let us assume that $f_i(\cdot)$ is a continuous and concave function of the differences in utilities promised by the two candidates' platforms:

$$\pi_{1i} = f_i(U_{1i} - U_{2i}), \qquad \pi_{2i} = 1 - \pi_{1i}. \tag{12.6}$$

Consider now a competition for votes between the two candidates defined over a policy space that consists simply of the distribution of Y dollars among the n voters.[2] Each voter's utility is a function of his income, $U_i = U_i(y_i)$, $U_i' > 0$, $U_i'' < 0$. Candidate 1 chooses a vector of incomes ($y_{11}, y_{12}, \ldots, y_{1i}$, and so on) to maximize her expected vote, EV_1, subject to the total income constraint; that is, she maximizes

$$EV_1 = \sum_i \pi_{1i} = \sum_i f_i(U_i(y_{1i}) - U_i(y_{2i})) + \lambda\left(Y - \sum_i y_{1i}\right). \tag{12.7}$$

Candidate 2 chooses a vector of incomes that maximizes $1 - EV_1$, which is to say a vector that minimizes EV_1. If the $f(\cdot)$ and $U(\cdot)$ functions are continuous and strictly concave, both candidates will choose the same platforms. These platforms will in turn satisfy the following first-order conditions:

$$f_i'U_i' = \lambda = f_j'U_j', \qquad i, j = 1, n. \tag{12.8}$$

Each candidate equates the weighted marginal utilities of the voters with the weights (f_i'), depending on the sensitivity of a voter's voting for a candidate to differences in the utilities promised by the candidates. The greater the change in the probability of

[2] Coughlin (1984, 1986) has analyzed this problem.

voter i's voting for 1 in response to an increase in $U_{1i} - U_{2i}$, the higher the income promised to i by both candidates.

If the probabilistic response of all voters to differences in promised utilities were the same – that is, $f_i'() = f_j'()$ for all i, j, – then (12.8) simplifies to

$$U_i' = U_j' \text{ for all } i, j = 1, n. \tag{12.9}$$

This condition is the same one that must be satisfied to maximize the Benthamite social welfare function (SWF)

$$W = U_1 + U_2 + \cdots + U_i + \cdots + U_n. \tag{12.10}$$

Thus, when the probabilistic response of all voters to differences in the expected utilities of candidate platforms is the same, the competition for votes between the candidates leads them to choose platforms that maximize the Benthamite SWF.[3] When the probabilistic responses of voters differ, candidate competition results in the maximization of a weighted Benthamite SWF.

A reasonable alternative to the assumption that voter decisions depend on the *differences* in expected utilities from the candidates' platforms is that they depend upon the ratios of utilities, that is, that π_{1i} is of the form

$$\pi_{1i} = f_i(U_{1i}/U_{2i}). \tag{12.11}$$

Substituting (12.11) into (12.7), and recalling that $U_{1i} = U_{2i}$ at the equilibrium, we obtain

$$f_i' \frac{U_i'}{U_i} = \lambda = f_j' \frac{U_j'}{U_j}, \qquad i, j = 1, n \tag{12.12}$$

as the first-order conditions for expected vote maximization for each of the candidates. When the marginal probabilistic responses are identical across all voters, this simplifies to

$$\frac{U_i'}{U_i} = \frac{U_j'}{U_j}, \qquad i, j = 1, n, \tag{12.13}$$

which is the first-order condition obtained by maximizing the Nash SWF

$$W = U_1 \cdot U_2 \cdot U_3 \cdots U_n. \tag{12.14}$$

Once again, candidate competition is seen to result in the implicit maximization of a familiar SWF.[4]

As a final example, consider again the spatial competition example with the three voters depicted in Figure 12.1. Let us assume that the probabilities of i supporting candidates 1 and 2 are defined by (12.6). Since we know this problem is equivalent to the maximization of (12.10), we can find the equilibrium platform that maximizes (12.10). We write the three voters' utility functions as $U_a = Z_a - (1-x)^2 - (1-y)^2$, $U_b = Z_b - (5-x)^2 - (1-y)^2$, $U_c = Z_c - (3-x)^2 - (5-y)^2$, where

[3] Ledyard (1984) obtains the Benthamite SWF using an assumption analogous to (12.6).

[4] Coughlin and Nitzan (1981a) obtain the Nash SWF from an assumption about the π_is analogous to (12.11).

the Z_is represent the utility levels achieved at each voter's respective ideal point. The two first-order conditions are

$$\begin{aligned} &2(1-x)+2(5-x)+2(3-x)=0 \\ &2(1-y)+2(1-y)+2(5-y)=0, \end{aligned} \tag{12.15}$$

from which we obtain the expected vote-maximizing platform for both candidates (3, 7/3), the point M in Figure 12.1. Competition for votes does drive the two candidates into the Pareto set to a point in the middle of the triangle.

When one assumes that the probabilities of voter support depend on differences in expected utility, competition drives candidates toward the (weighted) arithmetic mean of the voters' utilities. When the probabilities depend on ratios of utilities, the equilibrium is driven toward the geometric mean. Still other assumptions about the relationship between the probability of a voter's support and his expected utility under the competing platforms would produce equilibria at still other points. But as long as the probability of winning an individual's vote responds positively to increases in the voter's utility from a candidate's platform, then equilibria can be expected to be found within the Pareto set, and thus have desirable normative properties (Coughlin, 1982, 1992).

12.4 Equilibria with interest groups

The previous section describes a set of results under the probabilistic voting assumption that are indeed salutary. Political competition can produce equilibrium outcomes, and these outcomes can have potentially attractive normative properties. In this section we discuss an extension to the probabilistic voting model that sheds additional light on the nature of the outcomes obtained.

Coughlin, Mueller, and Murrell (1990) have extended the probabilistic voting model to allow for the impact of interest groups on political competition. Interest groups are defined as groups of individuals with identical tastes and incomes. If U_{ij} is the utility function of voter j who is a member of interest group i, then $U_{ij} = U_i$, for all $j = 1, n_i$, where n_i is the size of the ith interest group. Each individual is a member of one interest group.

The deterministic voting assumption (12.2) is replaced with the following assumption:

$$\begin{aligned} &(\pi_{1ij}=1) \leftrightarrow (U_{1i} > U_{2i} - b_{ij}) \\ &(\pi_{1ij}=0) \leftrightarrow (U_{1i} \le U_{2i} - b_{ij}) \\ &(\pi_{2ij}=1) \leftrightarrow (U_{1i} < U_{2i} - b_{ij}). \end{aligned} \tag{12.16}$$

The b_{ij} are "bias" terms. A $b_{ij} > 0$ implies a positive bias in favor of candidate 1 on the part of the jth voter in the ith interest group. The utility this voter expects from candidate 2's platform must exceed that expected from 1's platform by *more* than b_{ij}, before 1 loses this individual's vote to 2.

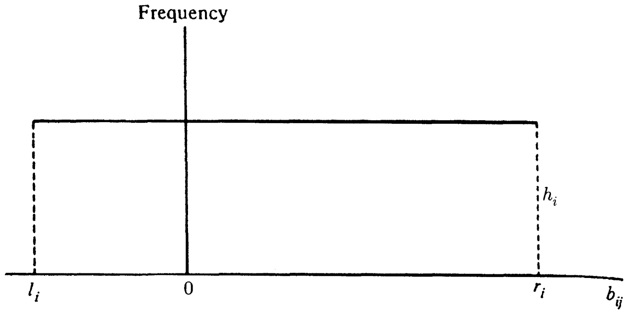

Figure 12.4. A uniform distribution of biases.

A probabilistic element is introduced into the model by assuming that the bias terms are random variables drawn from a probability distribution with parameters known to both candidates. Figure 12.4 depicts a uniform probability distribution for an individual in a given interest group. This group can be said to be biased in favor of candidate 1, since the bulk of the distribution lies to the right of the zero bias line. Nevertheless, some members of this group will be associated with negative bias terms. If candidate 1 matches 2's platform, she wins most but not all of the votes of interest group i.

The assumption that interest groups are biased toward or away from certain candidates or parties accords with observed voting patterns. Whites in the South and blacks everywhere in the United States tend to vote Democratic. Yankee farmers tend to vote Republican. On the other hand, not every Yankee farmer votes Republican.

The assumption that candidates know the distributions of bias terms, but not the individual bias term, implies that neither candidate can say with certainty how a given member of a particular interest group will vote. What they can predict is that they will pick up a greater fraction of an interest group's vote, the greater the difference in the utility their platform promises the representative interest group member over that of their opponent.

Assumption (12.16) makes the probability of i's supporting candidate 1 dependent on the difference between the utilities promised by the platforms of the two candidates. The first-order condition for expected vote maximization is thus of the form in (12.8). When the biases are drawn from the uniform distribution, however, f_i', the change in probability of winning the vote of a member of interest group i, is just the height of the uniform distribution, h_i, from which the b_{ij} are drawn, since the area of the uniform distribution equals one, $h_i = 1/(r_i - l_i)$. Thus, under the assumption that the bias terms are uniformly distributed, two-candidate competition for votes leads each candidate to offer platforms that maximize the following welfare function:

$$W = \alpha_1 n_1 U_1 + \alpha_2 n_2 U_2 + \cdots + \alpha_m n_m U_m, \qquad (12.17)$$

where the $\alpha_i = f_i' = 1/(r_i - l_i)$. The greater the difference between r_i and l_i, the boundaries on the uniform distribution for interest group i, the greater the range over which the b_{ij} are distributed. The greater this range, the more important the

b_{ij} become in determining how an interest group's membership votes, and the less important the promised utilities are. Given the latter, both candidates give less weight to this group's interests in choosing platforms.

The results from this probabilistic voting model with interest groups resemble those of the earlier models in that equilibria exist and are Pareto optimal. In fact, an additive welfare function is maximized, albeit one that assigns different weights to the different interest groups.

This latter property raises important normative issues about the equilibria obtained in the competitive struggle for votes. Although candidates are uncertain about how the members of different interest groups will vote, they are uncertain in different degrees about different groups. One way in which interest groups attempt to influence public policy is to make candidates aware of potential votes to be won from their interest group by taking certain positions in their platforms. Interest groups try to increase the welfare of their membership by reducing candidate uncertainty over how their membership votes.

But this in turn implies that different interest groups receive different weights in the candidates' objective functions and thus receive different weights in the social welfare function, which is implicitly maximized through candidate competition. When candidates are unsure of the votes of different groups, and these groups have different capabilities in approaching candidates, then one's benefits from political competition depend in part upon the interest group to which one belongs. The egalitarianism inherent in the slogan "one man, one vote" is distorted when interest groups act as intermediaries between candidates and citizens.

12.5 An application to taxation

12.5.1 *The logic*

Probabilistic voting models have become increasingly popular over the past 20 years for analyzing electoral politics. Much of the literature on interest groups has employed this model, for example, and we shall focus upon it in Chapter 20. Here we confine ourselves to a brief look at an application of the model to taxation.

Let us imagine a country with a two-party political system. The economy has one private good, X, and the government supplies one public good, G, which it finances with taxes on individual incomes. We shall assume that the government can levy a separate tax, t_i, on each individual i. Each individual's income, Y_i, is devoted entirely to her own personal consumption of X and her tax payment, $Y_i = (1 - t_i)X_i$. Under these assumptions, the expected vote function of party 1, as given in (12.7), is modified to become

$$EV_1 = \sum_1 \pi_{1i} = \sum_i f_i(U_i(G, X_{1i}) - U_i(G, X_{2i})) + \lambda\left(\sum_i Y_i - G - \sum_i X_i\right). \qquad (12.18)$$

To balance its budget, the government must choose individual tax rates, t_i, such that $G = \sum_{i=1}^{n} t_i Y_i$. Party 1 maximizes its expected vote by choosing G and the t_i to maximize (12.18). Maximizing with respect to G yields the first-order condition

$$\sum_{i=1}^{n} f_i' \frac{\partial U_i}{\partial G} = \lambda. \tag{12.19}$$

Setting $G = \sum_{i=1}^{n} t_i Y_i$ in the budget constraint term of (12.18), substituting into each $U_i(G, X_i)$ from the individual budget constraints, and then maximizing with respect to t_i gives the following first-order conditions:

$$f_i' \frac{\partial U_i}{\partial X_i} = \lambda, \qquad i = 1, n. \tag{12.20}$$

A comparison of (12.19) and (12.20) with (2.8) and (2.9) from Chapter 2 reveals that they are the same except that we have now implicitly assumed that $P_G = P_X = 1$, and the γ_is from (2.8) and (2.9) have been replaced by f_i's. The γ_is in (2.8) and (2.9) were the positive weights placed on each individual's utility in the SWF (2.6) that was maximized to find the Pareto-optimal quantity of the public good. The f_i's are the weights that each party implicitly places on the utilities of each individual when it maximizes its expected vote. As was done in Chapter 2, each f_i' from (12.20) can be used to replace an f_i' in (12.19) to yield

$$\sum_i \frac{\partial U_i/\partial G}{\partial U_i/\partial X_i} = 1, \tag{12.21}$$

where (12.21) is again the Samuelsonian (1954) condition for Pareto optimality in the presence of public goods when $P_G = P_X$. Although each party is only interested in maximizing its expected votes, the competition for votes forces each to choose individual taxes and a public good quantity that satisfies the conditions for Pareto optimality.

Although the outcome of electoral politics from the probabilistic voting model satisfies the condition for Pareto optimality, the realized utility levels implied by (12.19) and (12.20) are possibly quite different from those that an impartial social planner might induce by selecting a set of γs for his SWF. Equation (12.19) implies that the political process produces a large quantity of the public good if the votes of those who favor large quantities of the public good are highly responsive to the announced platforms of the parties (their $f'(\cdot)$s are large). Equation (12.20) states that individuals whose votes are highly responsive to the announced platforms of the parties are left with command over larger quantities of the private good (are assigned low taxes).

This comparison of the first-order conditions that one obtains by maximizing an SWF, and the first-order conditions that are implicitly obtained through the process of electoral competition, reveals a perhaps surprising similarity between the predictions for tax policy that emerge from a *positive* analysis of taxation using the probabilistic voting model, and the *normative* prescriptions that one derives from optimal tax theory. Both imply, for example, the potential for a highly complex tax structure. When individual utility functions differ greatly and yet all must consume the same quantities of public goods, assigned tax prices may have to differ greatly to satisfy the first-order conditions for Pareto optimality. When individuals differ greatly in their access and responsiveness to politics, parties may be forced to offer individuals and groups greatly different tax prices if the parties wish to maximize their chances of getting elected.

These predictions from the positive analysis of taxation differ greatly from the normative prescriptions of scholars like Simons (1938) and most recently Buchanan and Congleton (1998), who argue that the equitable treatment of individuals requires that citizens in similar situations be taxed similarly.[5] Despite the many advocates of such forms of horizontal equity, and the many proposals for broad-based and "flat" taxes, the tax code in the United States and most other developed countries remains a thicket of exemptions and special privileges. Thus, this prediction of the positive theory seems, from casual observation, to be borne out. We turn now to some more systematic evidence regarding the determinants of tax structure.

12.5.2 *The evidence*

The probabilistic voting model predicts that tax policy is slanted in favor of persons and groups who are able to deliver votes to a party that offers them favorable tax treatment. To test the model one needs to identify the persons or groups with the greatest capacities for delivering votes, and test to see whether they receive favorable treatment in the tax structure. Since no indexes of political strength are readily available, the probabilistic voting model does not immediately lead to strong predictions as to which specific groups are going to receive favorable tax treatment.

A second difficulty in testing the implications of the probabilistic voting model arises because it makes some of the same predictions as its competitors. For example, a major result in the optimal tax literature is that tax policy should attempt to minimize deadweight losses. A vote-maximizing party will also be interested in containing deadweight losses, however, because they cause it to lose votes. Indeed, the optimal set of taxes from the point of view of a vote-maximizing party – as for the welfare function maximizing social planner – would be a set of lump-sum taxes. The two ideal policies would differ not in the *form* that the taxes would take, but rather in their magnitudes. Thus, evidence like that presented by Kenny and Toma (1997), that tax and seigniorage policy in the United States over time has tended to smooth income, as the optimal tax literature says it should, is also consistent with the hypothesis that these policies are introduced by parties seeking to maximize votes in elections.[6]

The most obvious alternative to the probabilistic voting model for explaining tax policy is the median voter model. But here, too, the two models may lead to similar predictions, if it is reasonable to assume that the middle class is an effective political group (has a high f' in (12.20)). Does the existence of tax deductions for children

[5] See the discussion by Hettich and Winer (1999, ch. 5).

[6] The same, of course, can be said for many of the other empirical studies that attempt to test propositions from optimal tax theory. See references in Kenny and Toma (1997) and Hettich and Winer (1999, ch. 8).

imply that parents are a politically effective interest group, that the median voter has children, or that the social planner has placed extra weight on the utility functions of people with children?

Despite these conundrums, in some cases it is possible to infer that an observed pattern of taxes is consistent with certain groups exercising greater influence in the determination of taxes. For example, owners of expensive houses are not likely to get extra weight in a reasonable social planner's welfare function nor to include the median voter in their group. Hunter and Nelson's (1989) finding that the share of total tax revenue in Louisiana parishes accounted for by property taxes is inversely related to the percent of the homeowners who own expensive houses, thus seems to confirm their hypothesis that these wealthy homeowners are an effective political group in Louisiana.[7]

Hettich and Winer (1984, 1999, ch. 9) employ the probabilistic voting model to motivate their study of the reliance on the income tax as a source of revenue across states. The clearest support for the probabilistic voting model actually comes from the second equation in their model, which predicts whether a state allows residents to credit their property tax payments against their state income tax obligations. Once again wealthy homeowners appear to exert significant political influence as do citizens over 65.[8]

Although the number of studies that directly test for the importance of political strength in determining tax structure is small, the results so far are encouraging.

12.6 Commentary

When Anthony Downs put forward his economic theory of democracy, he seemed to suggest that the outcomes from a political system in which candidates competed for the votes of the electorate would somehow avoid the nihilistic implications of the cycling literature, and more generally Arrow's impossibility theorem (see, e.g., Downs, 1957, pp. 17–19). Downs did not succeed in demonstrating any normative results concerning the outcomes from political competition, however, and the subsequent literature on spatial voting models proved in one paper after another that cycling is potentially just as big a problem when candidates compete for votes as it is for committee voting.

The literature on probabilistic voting appears to drive a giant wedge between the public choice literature on committee voting and that on electoral competition. Committee voting is inherently deterministic, and cycling problems will continue to confound the outcomes from committee voting under rules like the simple majority rule. But if voters reward a candidate who promises them a higher utility by increasing the likelihood of voting for the candidate, then competition for votes between candidates leads them "as if by an invisible hand" to platforms that maximize social welfare. The analogy between market competition and political competition does

[7] A parish in Louisiana is the local political unit corresponding to the county in other states. Farmers were also identified as an effective political group by Hunter and Nelson.

[8] Many additional variables, which Hettich and Winer hypothesize will be significant, prove to be so. But often these other variables might also be consistent with alternative models.

exist. Both result in Pareto-optimal allocations of resources. Downs's faith in the efficacy of political competition has at long last been vindicated.

Several writers have questioned the reasonableness of some of the assumptions upon which the main theorems in the probabilistic voting literature rest, namely, that the probability functions of a voter voting for a given candidate are monotonically increasing and concave in the utility promised to the voter by the candidate, and the issue set over which the candidates compete is compact and convex (Slutsky, 1975; Usher, 1994; Kirchgässner, 2000).

Kirchgässner, for example, questions the generality of the probabilistic voting models by constructing an example for three voters with ideal points located to form a triangle as in Figure 12.1. He then chooses probabilities such that candidate 2 can increase her chances of winning the votes of A and B by moving to the midpoint of $\overline{AB}$ by more than enough to offset the reduction in the probability of C voting for her, assuming candidate 1 is located at M. Thus Kirchgässner argues cycling can also arise with probabilistic voting.

Clearly, a three-voter electorate is a rather unusual assumption and it might be reasonable to assume that candidates hop about trying to win the votes of two of the three voters. With a large number of voters and a unimodal distribution of ideal points, such jumping around with probabilistic voting would seem much less reasonable. Even with a three-voter electorate, however, the theorems proving the existence of equilibria under probabilistic voting remain valid – if one maintains the assumptions of the theorems.

In their proofs of the existence of an equilibrium under probabilistic voting, Coughlin and Nitzan (1981a,b) assume that the probability of voter i voting for each of the two candidates is a concave function of the following form:

$$\pi_{1i} = \frac{U_{1i}}{U_{1i} + U_{2i}}, \qquad \pi_{2i} = \frac{U_{2i}}{U_{1i} + U_{2i}}. \tag{12.22}$$

Now assume that each voter i's utility from the platform of candidate j takes the following form:

$$U_i^j = K - \mid I_i - P_j \mid^2, \tag{12.23}$$

where I_i is voter i's ideal point, P_j is the platform of candidate j, and $\mid I_i - P_j \mid$ is the Euclidean distance between the two points. K is a positive constant that represents the utility each voter experiences from an $x - y$ combination located at his ideal point. K must be sufficiently large to make $U_i^j > 0$, if it makes sense to provide the public goods x and y at all.

If candidate 1 locates at M, equidistant from A, B, and C, and candidate 2 is halfway between A and B, then the probability of candidate 1 getting the vote of either A or B is

$$\pi_{1A} = \pi_{1B} = \frac{K - \left(\frac{2}{\sqrt{3}}\right)^2}{K - \left(\frac{2}{\sqrt{3}}\right)^2 + K - 1} = \frac{K - \frac{4}{3}}{2K - \frac{7}{3}}, \tag{12.24}$$

while the probability of getting C's vote is

$$\pi_{1C} = \frac{K - \frac{4}{3}}{K - \frac{4}{3} + K - 3} = \frac{K - \frac{4}{3}}{2K - \frac{13}{3}}. \tag{12.25}$$

The respective probabilities for candidate 2 are

$$\pi_{2A} = \pi_{2B} = \frac{K - 1}{2K - \frac{7}{3}}, \qquad \pi_{2C} = \frac{K - 3}{2K - \frac{13}{3}}. \tag{12.26}$$

Summing each probability function over the three voters we obtain π_1 and π_2, from which it is easy to show that

$$(\pi_1 > \pi_2) \longleftrightarrow \left(K > \frac{1}{2}\right). \tag{12.27}$$

Recalling that K must be sufficiently large to make the provision of x and y to the community worthwhile, it is easy to see that (12.27) is satisfied for each of the platforms of the two candidates. Candidate 2 does not increase her probability of winning by leaving point M.

If we think of the two candidates as promising different bundles of public goods, then the imposition of a budget constraint on the government or a resources constraint on the economy would suffice to make the issue set satisfy the compactness and convexity assumption. With two public goods, x and y, and a budget constraint, B, the condition is satisfied. Are these reasonable assumptions? Is there a finite probability that a given citizen will vote for candidate 1 for every possible platform this candidate might choose? Do these platforms range to infinity in some directions of the issue space? Ultimately, these are questions about the voter's psychology that cannot be resolved by logical argument.[9]

An alternative to testing the accuracy of the assumptions underlying the theorems is, of course, to test their implications. In a two-party system like that of the United States do the candidates seem to converge on the same (similar) positions on the full set of issues? Do the outcomes of the electoral process sometimes produce candidates who take extreme positions on one set of issues, and other times take them on a totally different set? If the reader thinks that this is the case, then she should be skeptical of the assumptions underlying the probabilistic voting models. If she does not, she can take some comfort in their implications.

Even if we accept the underlying assumptions of the probabilistic voting models and their implications about equilibria under two-party competition, they can raise additional normative issues of a less salutary nature. The probabilistic voting model

[9] Enelow and Hinich (1989) introduce a probabilistic element in a two-party electoral model as a random error term in a candidate's expectation of her share of the vote. Whether an equilibrium exists or not is shown to depend on "the variance of the random element . . . , the size of the feasible set of candidate policy locations, the salience of policies among voters, the dimensionality of the policy space, and the degree of concavity in voter utility functions" (p. 110). Thus, Enelow and Hinich's probabilistic voting model illustrates some of the points Kirchgässner makes in his critique. The existence of an equilibrium is not guaranteed by the introduction of a random element in the two-party model. Once again, however, it is not easy to say whether the assumptions about the size of the feasible set, concavity of voter utility functions, and so on needed to ensure an equilibrium are reasonable or not.

with interest groups implies that different groups receive different weights in the welfare function, which candidate competition implicitly maximizes. The empirical literature on taxation discussed earlier and that reviewed in Chapter 20 underscore the importance of this issue by providing ample evidence of a two-way exchange relationship between candidates and interest groups. While it is comforting to know that political competition takes us to an equilibrium on the Pareto-possibility frontier, before we sing the praises of two-party democracy too loudly we might wish to inquire about where this point on the frontier lies. Before passing judgment on the merits of a two-party system, it also might be prudent to compare it to its alternatives – one-party and multiparty systems. We take up multiparty systems in the next chapter, and leave single-party systems for Chapter 18.

Bibliographical notes

The first articles to establish the existence of equilibria under probabilistic voting assumptions were by Davis et al. (1970) and Hinich, Ledyard, and Ordeshook (1972, 1973). Although the equilibrium result was clearly there, the significance of the result was not appreciated by this observer, because the probabilistic element in the models was assumed to be due to abstentions when candidates were too far from a voter's ideal point. Thus, equilibria appeared to emerge as a sort of accidental consequence of some voters refusing to vote. This seemed a shaky foundation upon which to build a strong normative case for the outcomes from electoral competition. As the literature has evolved, however, the emphasis has shifted from abstentions to uncertainty on the part of candidates and/or voters. Relevant papers in this evolution include Comanor (1976), Denzau and Kats (1977), Hinich (1977), Coughlin and Nitzan (1981a,b), Coughlin (1982, 1984, 1986), and Ledyard (1984). Enelow and Hinich (1984, ch. 5), Ordeshook (1986, pp. 177–80; 1997), and Coughlin (1992) provide overviews of this literature.

The normative significance of the results is brought out most clearly by Coughlin and Nitzan (1981a), Coughlin (1982, 1984, 1992), and Ledyard (1984) and stressed most forcefully by Wittman (1989, 1995).

Wittman (1984) extends the equilibrium results to competition among three or more candidates, Austen-Smith (1981b) to multiconstituency party competition.

Samuelson (1984) assumes that candidates begin at different starting points and are constrained in how far from these starting points they can move in any election. Equilibria occur with the candidates adopting different platforms and having different expected vote totals. Hansson and Stuart (1984) obtain similar results by assuming that candidates have utility functions defined over the strategy choices.

The public choice analysis of taxation was launched by Hettich and Winer (1984, 1988), who have also surveyed the major contributions to the literature (Hettich and Winer, 1997, 1999).

Finally, mention must be made of an important related work of Becker (1983). Becker does not model the process of political competition, but assumes that government is a form of market for equilibrating interest group demands for favors. At the assumed equilibrium, Pareto optimality holds as in the equilibria of the probabilistic voting models.